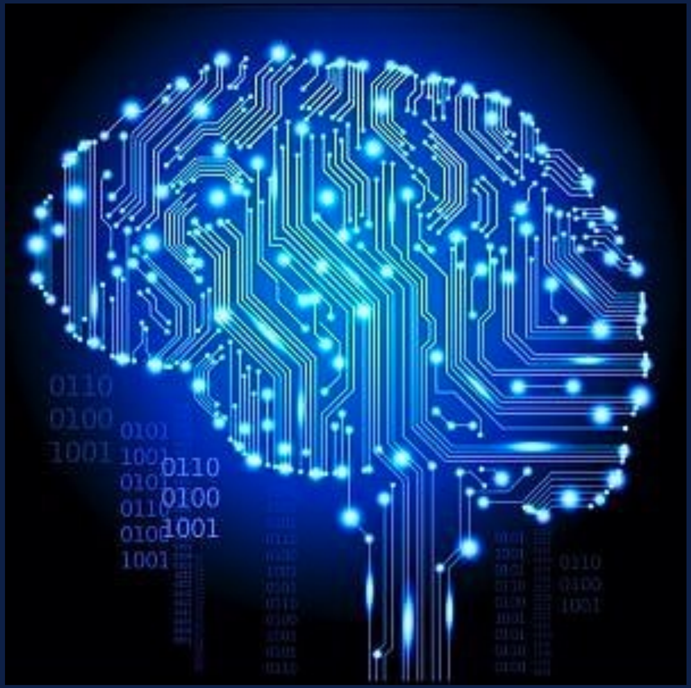
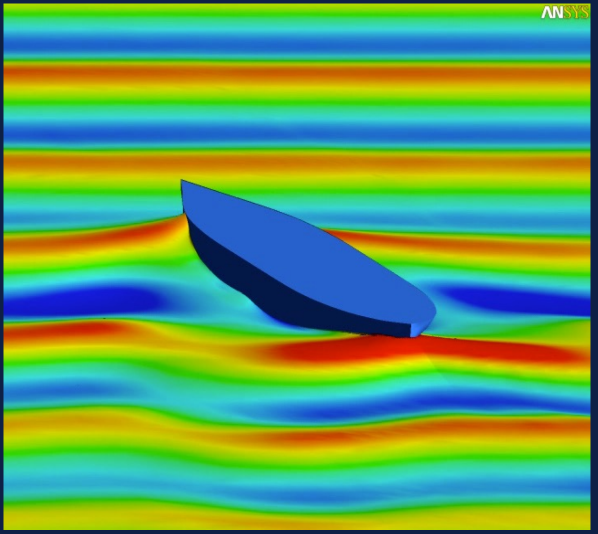
Intelligenza Artificiale (IA)

Parte 4 : Modelli Matematici (Physics Based)

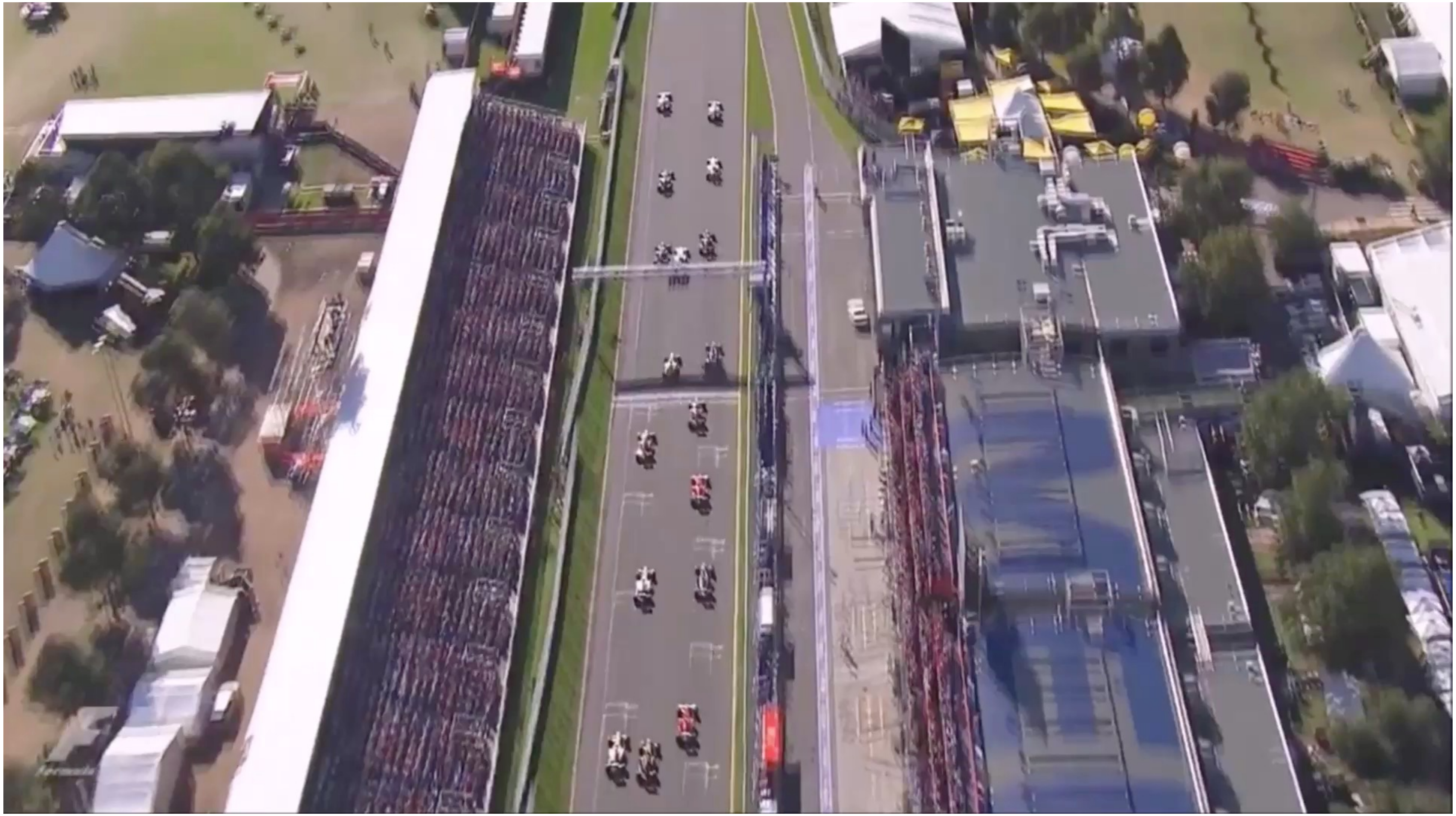


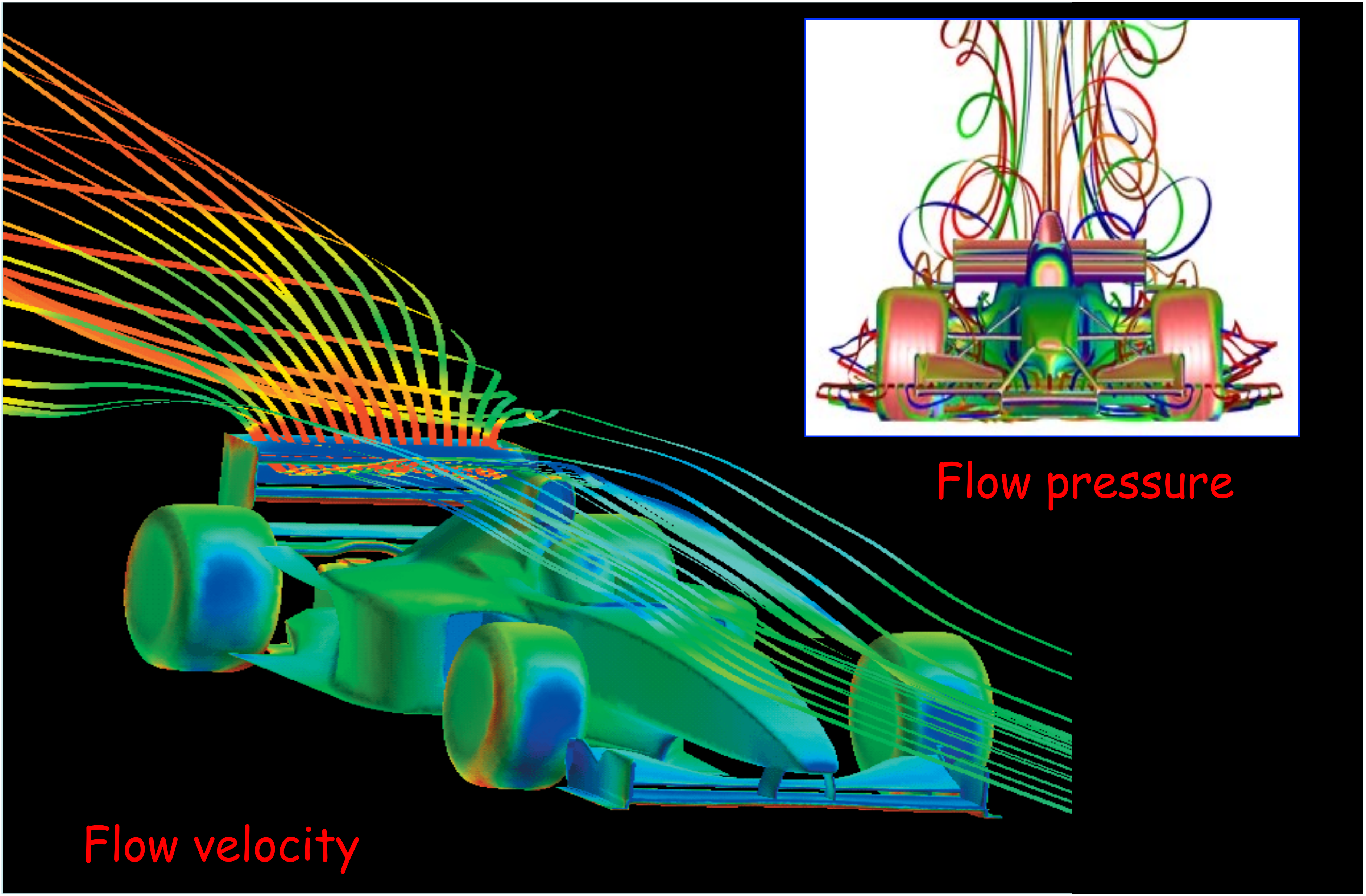
MATH for SPORTS



Alfio Quarteroni

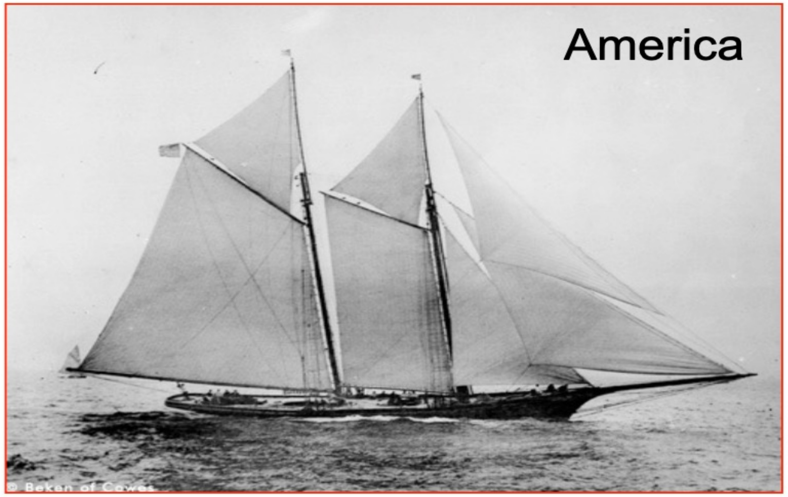
Politecnico di Milano - EPFL




Flow pressure
Flow velocity

A Brief History of the America's Cup

- Held by the U.S.A. for 132 years since 1851, the cup was then won sporadically by few other Countries (New Zealand and Australia)
- Switzerland competing for the first time in 2003 with EPFL engaged as scientific consultant


America

Alinghi SUI 64 - 2003

- The Swiss Team Alinghi won the 2003 and 2007 consecutive editions
- Switzerland still remains the only European country to have won the America's Cup

Alinghi SUI 100 - 2007

Our Main Achievements Thanks to Math Modelling

Math Model for America's Cup

Air Phase

$$\frac{\partial(\rho_a \boldsymbol{u}_a)}{\partial t} + \boldsymbol{\nabla}\cdot(\rho_a \boldsymbol{u}_a \otimes \boldsymbol{u}_a) - \boldsymbol{\nabla}\cdot \boldsymbol{T}_a(\boldsymbol{u}_a, p_a) = \rho_a \boldsymbol{g}$$

$$\boldsymbol{\nabla}\cdot \boldsymbol{u}_a = 0$$

Interface Conditions

$$\boldsymbol{u}_a = \boldsymbol{u}_w \quad \text{on } \Gamma$$

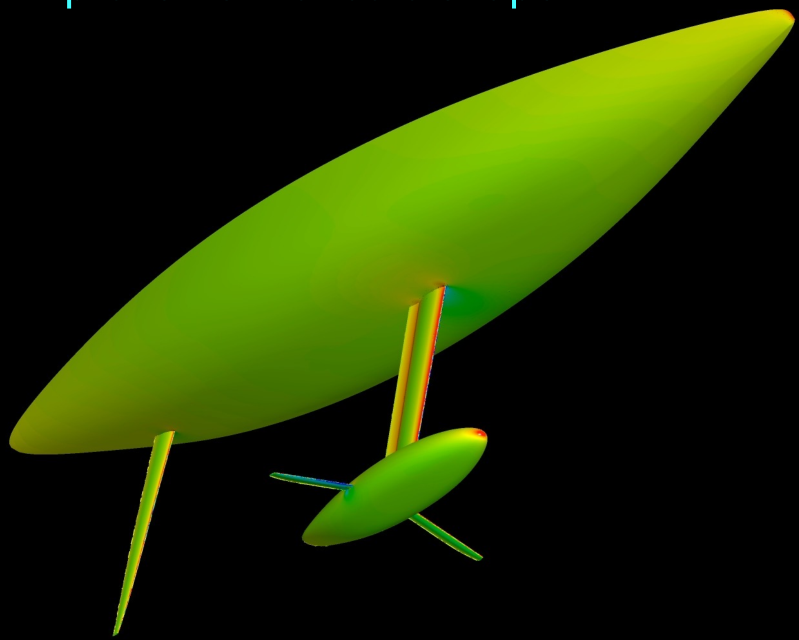
$$\boldsymbol{T}_a(\boldsymbol{u}_a, p_a)\cdot \boldsymbol{n} = \boldsymbol{T}_w(\boldsymbol{u}_w, p_w)\cdot \boldsymbol{n} + \kappa\sigma \boldsymbol{n} \quad \text{on } \Gamma$$

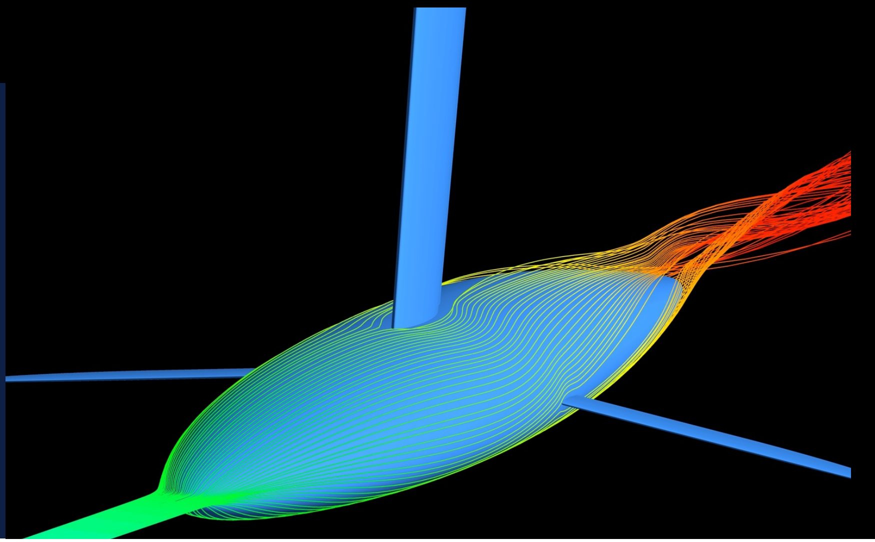
$$\frac{\partial(\rho_w \boldsymbol{u}_w)}{\partial t} + \boldsymbol{\nabla}\cdot(\rho_w \boldsymbol{u}_w \otimes \boldsymbol{u}_w) - \boldsymbol{\nabla}\cdot \boldsymbol{T}_w(\boldsymbol{u}_w, p_w) = \rho_w \boldsymbol{g}$$

$$\boldsymbol{\nabla}\cdot \boldsymbol{u}_w = 0$$

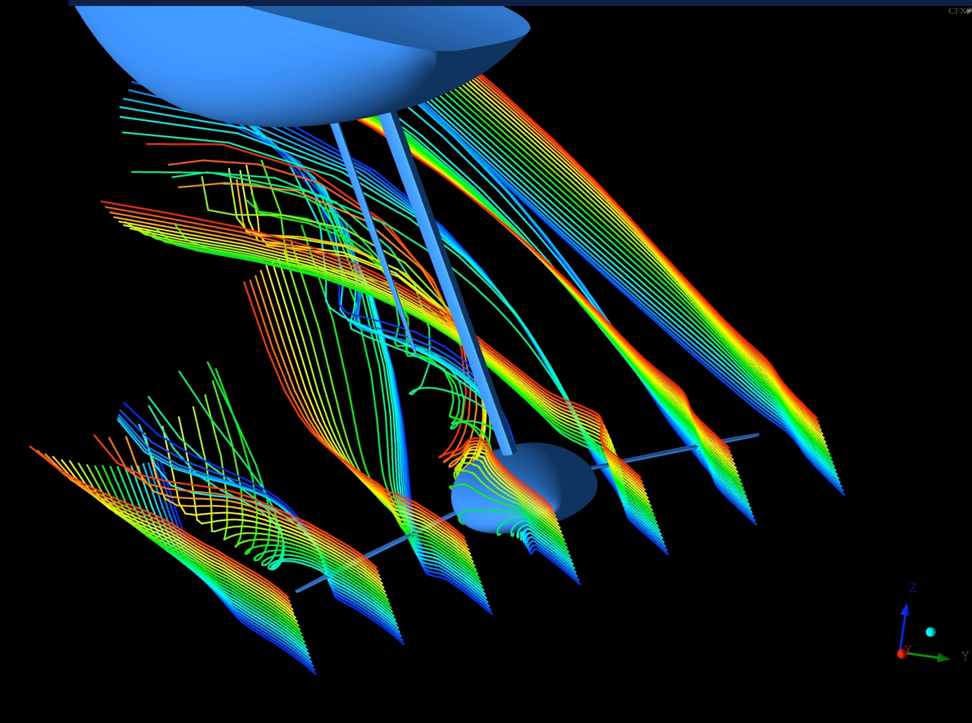
Water Phase

Improvement of bulb shape

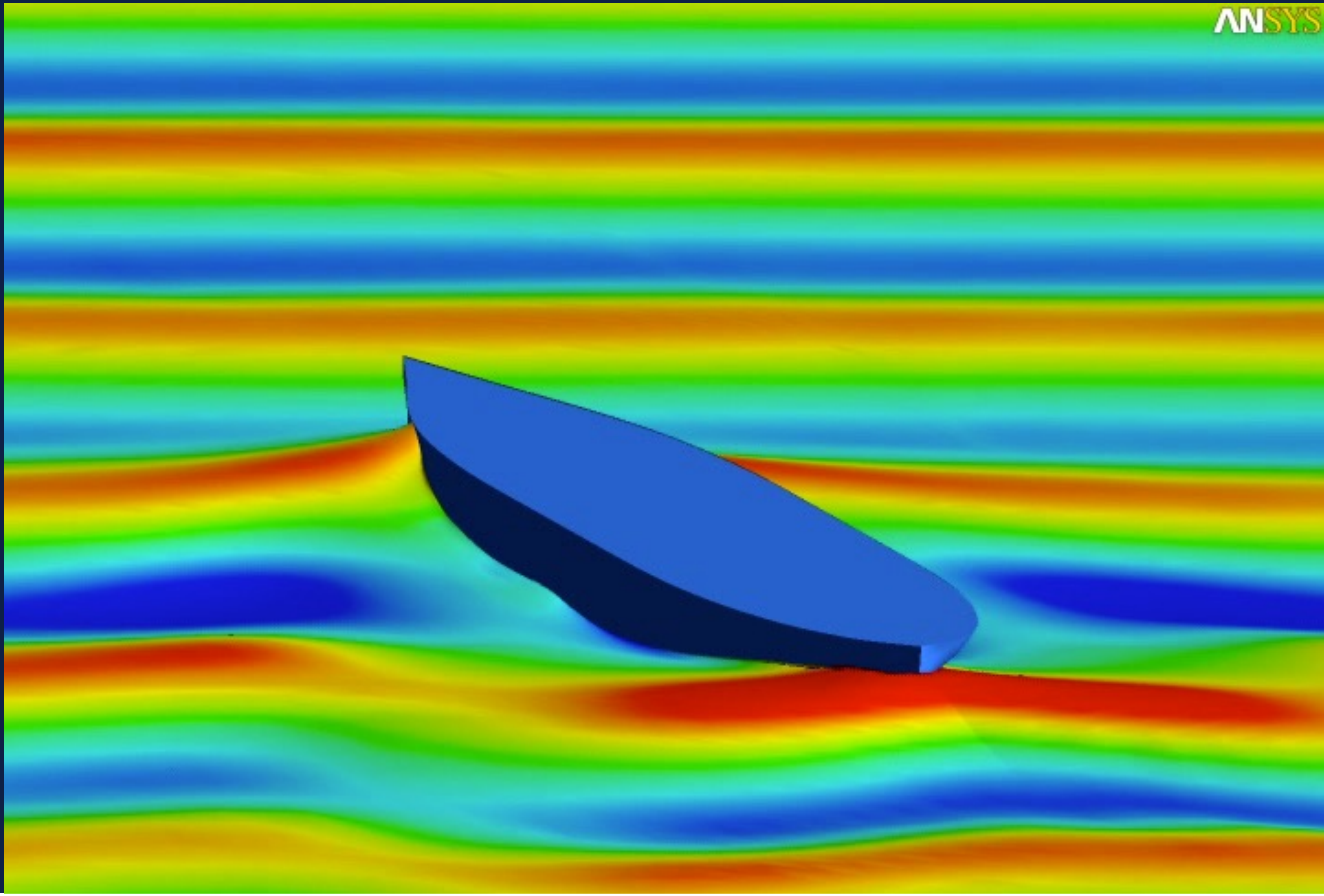




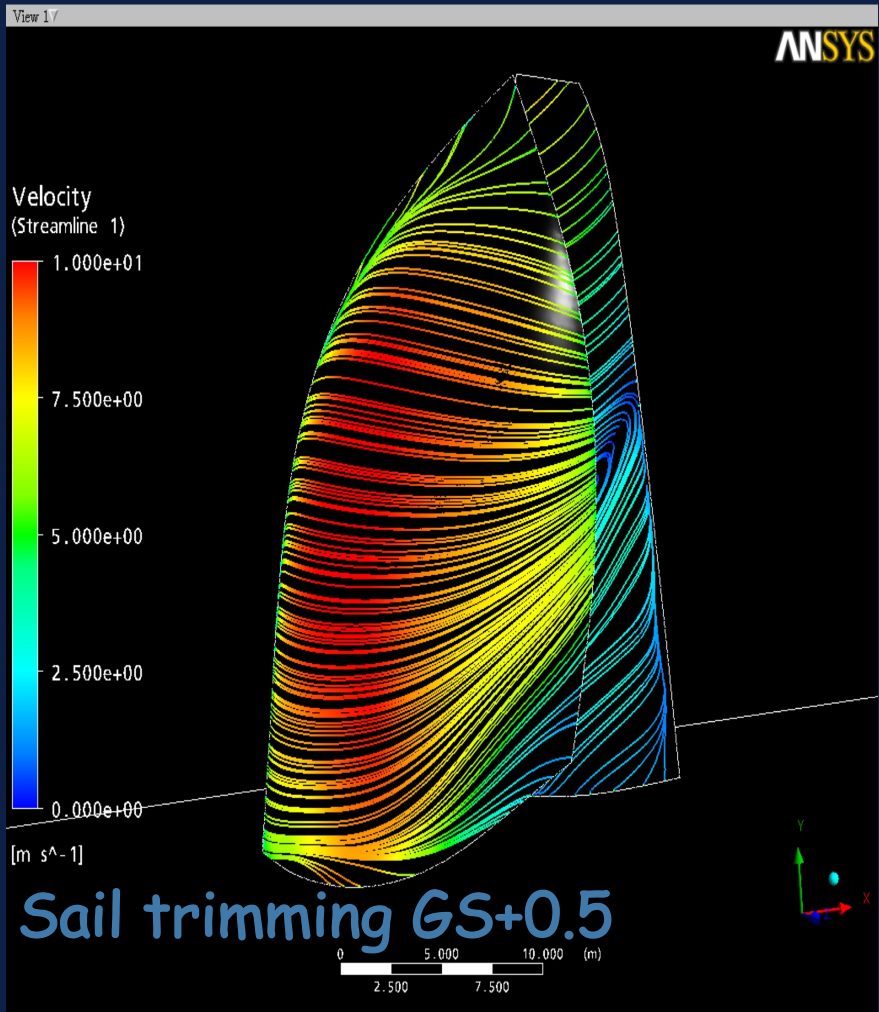
Better dislocation of winglets on the bulb



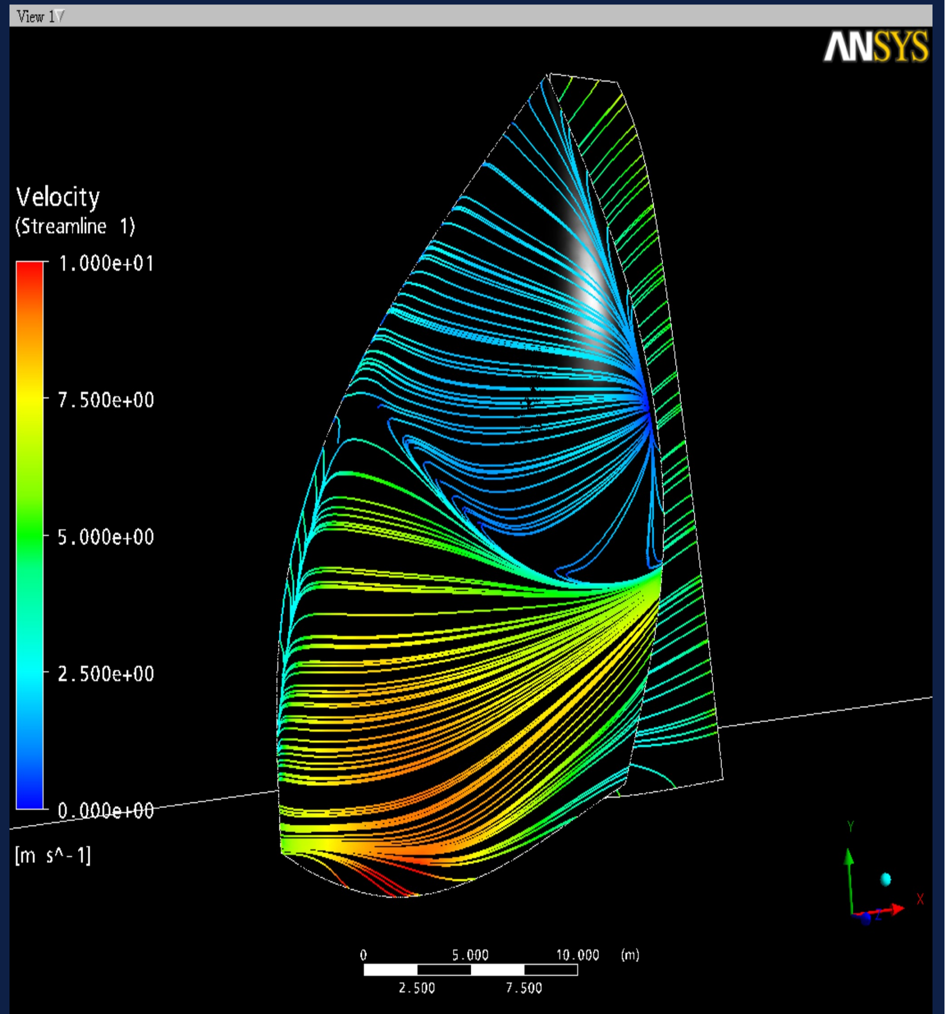
Minimisation of the wet surface on hydrostatic regime



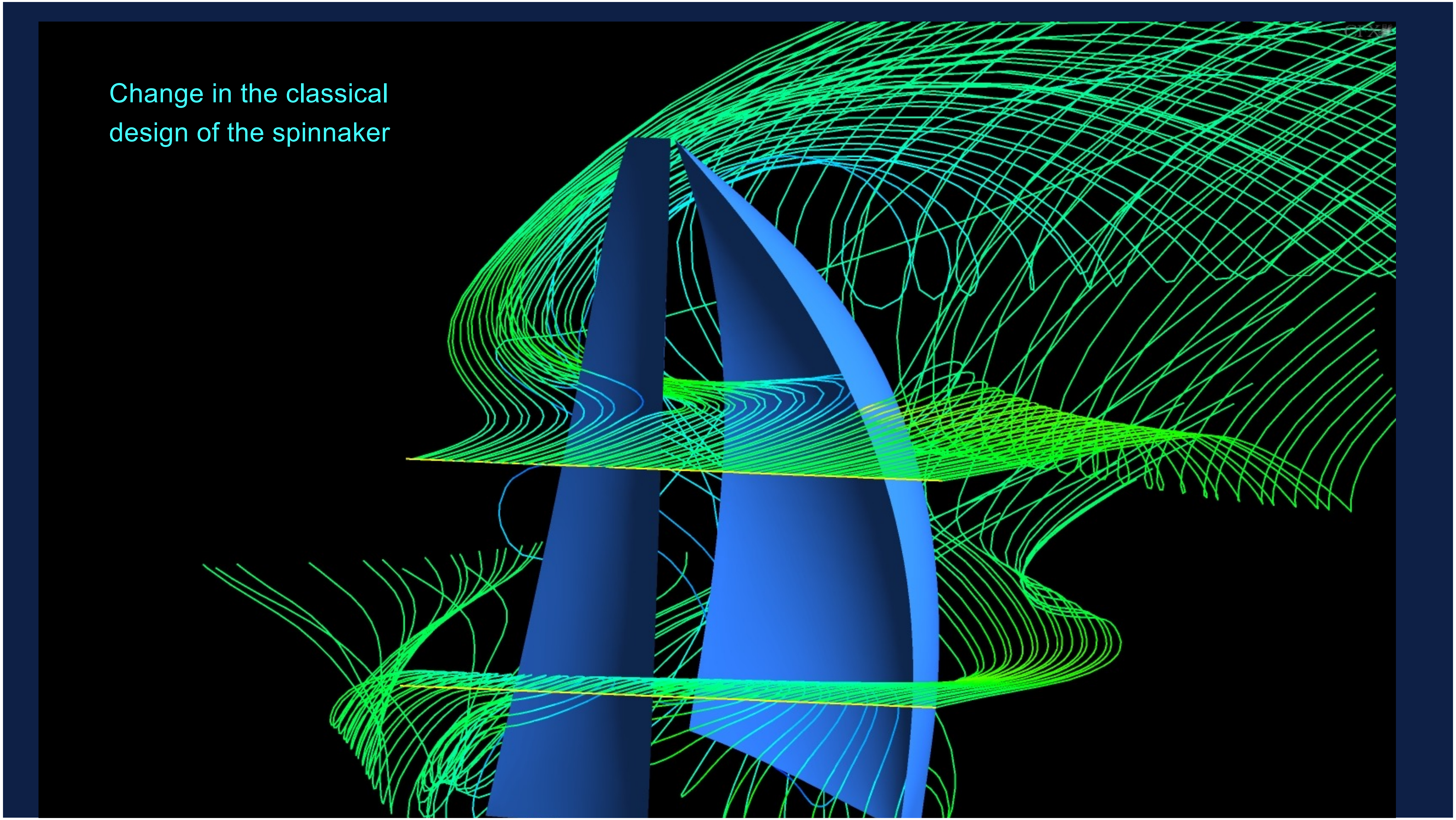
Sail trimming to maximize flow reattachment



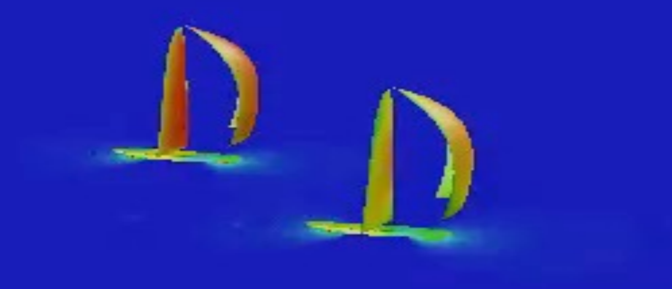
Sail trimming GS+0.5



Sail trimming GS-1

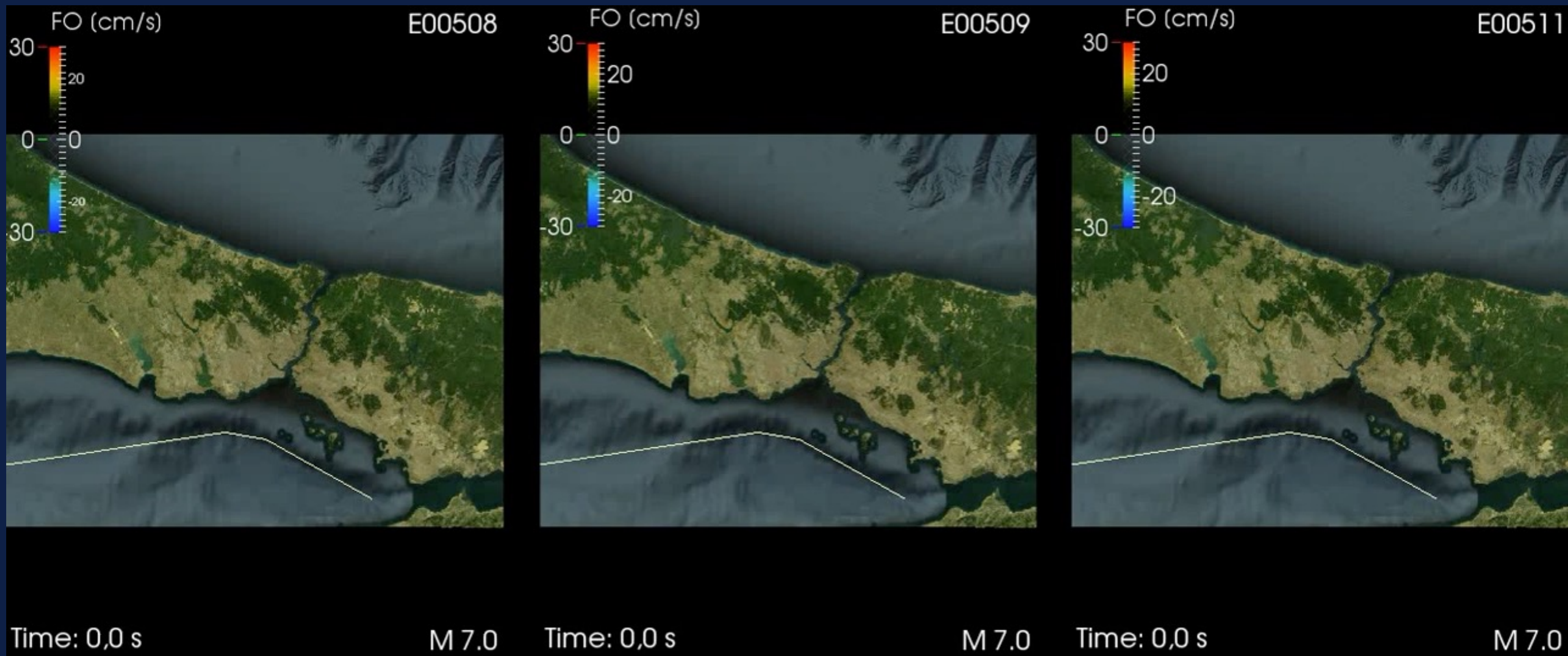

Change in the classical
design of the spinnaker

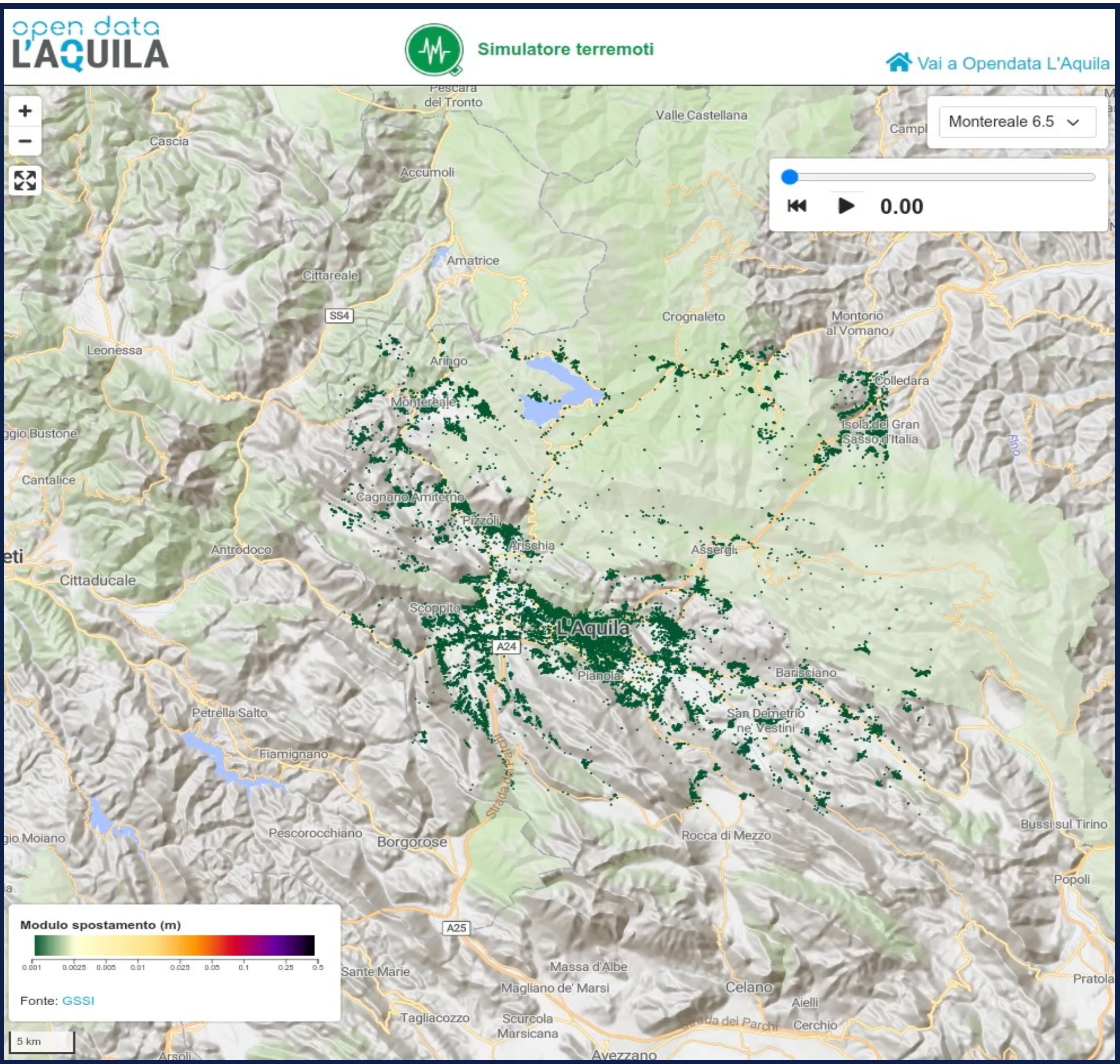
Maximise the shadow region in downstream leg



Alinghi

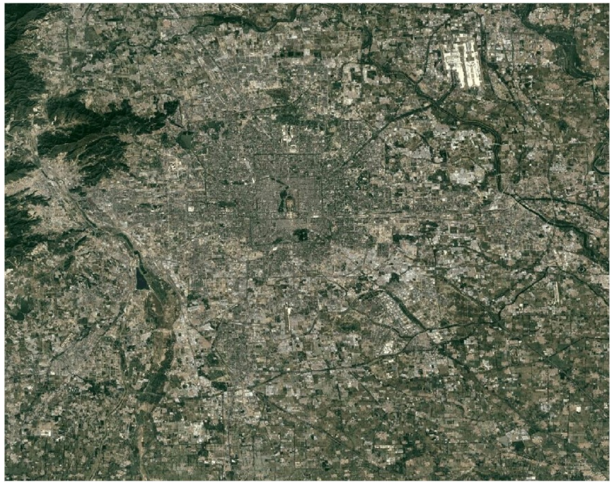
MATH for EARTHQUAKES




open data
L'AQUILA
Simulatore terremoti
Vai a Opendata L'Aquila
Montereale 6.5
0.00
Modulo spostamento (m)
Fonte: GSSI
L'Aquila

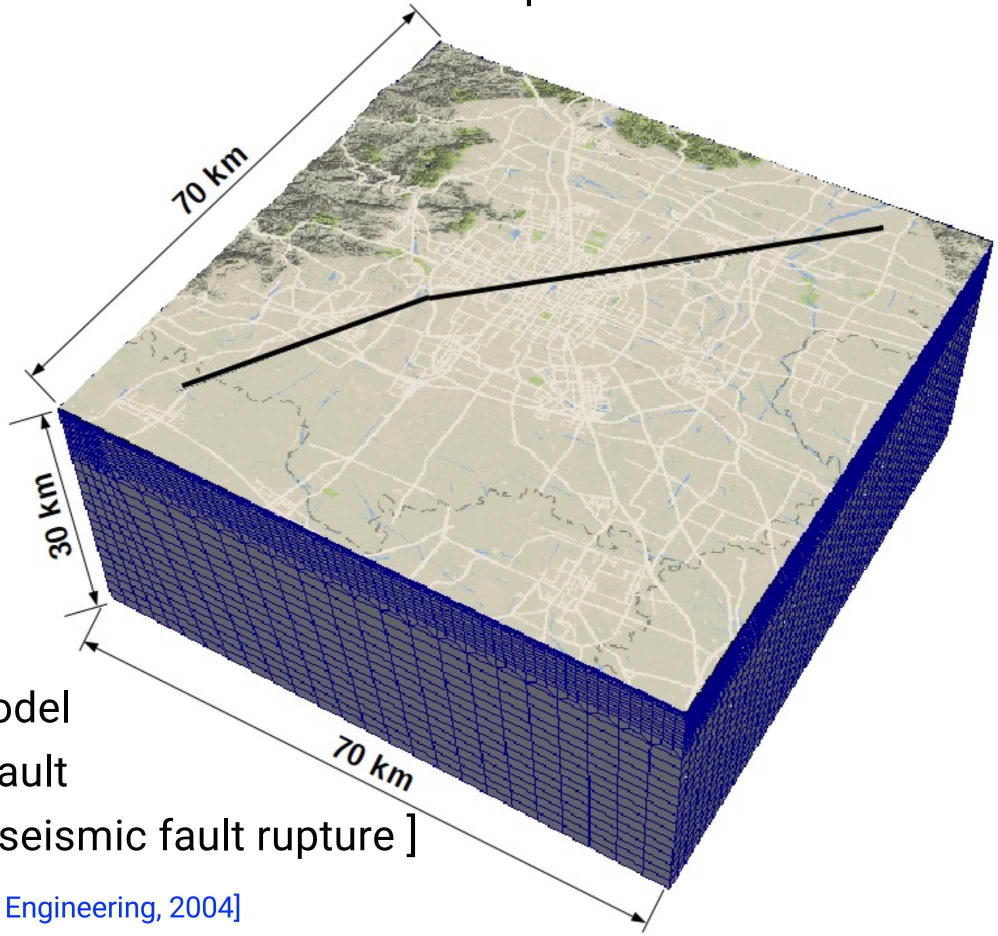
The Beijing metropolitan area

3D physics-based numerical modelling of Beijing



Beijing metropolitan area
Land: 16.801 km^2
Popolation: 21.707.000
Density: 1.300/km^2

Main features of the computational model



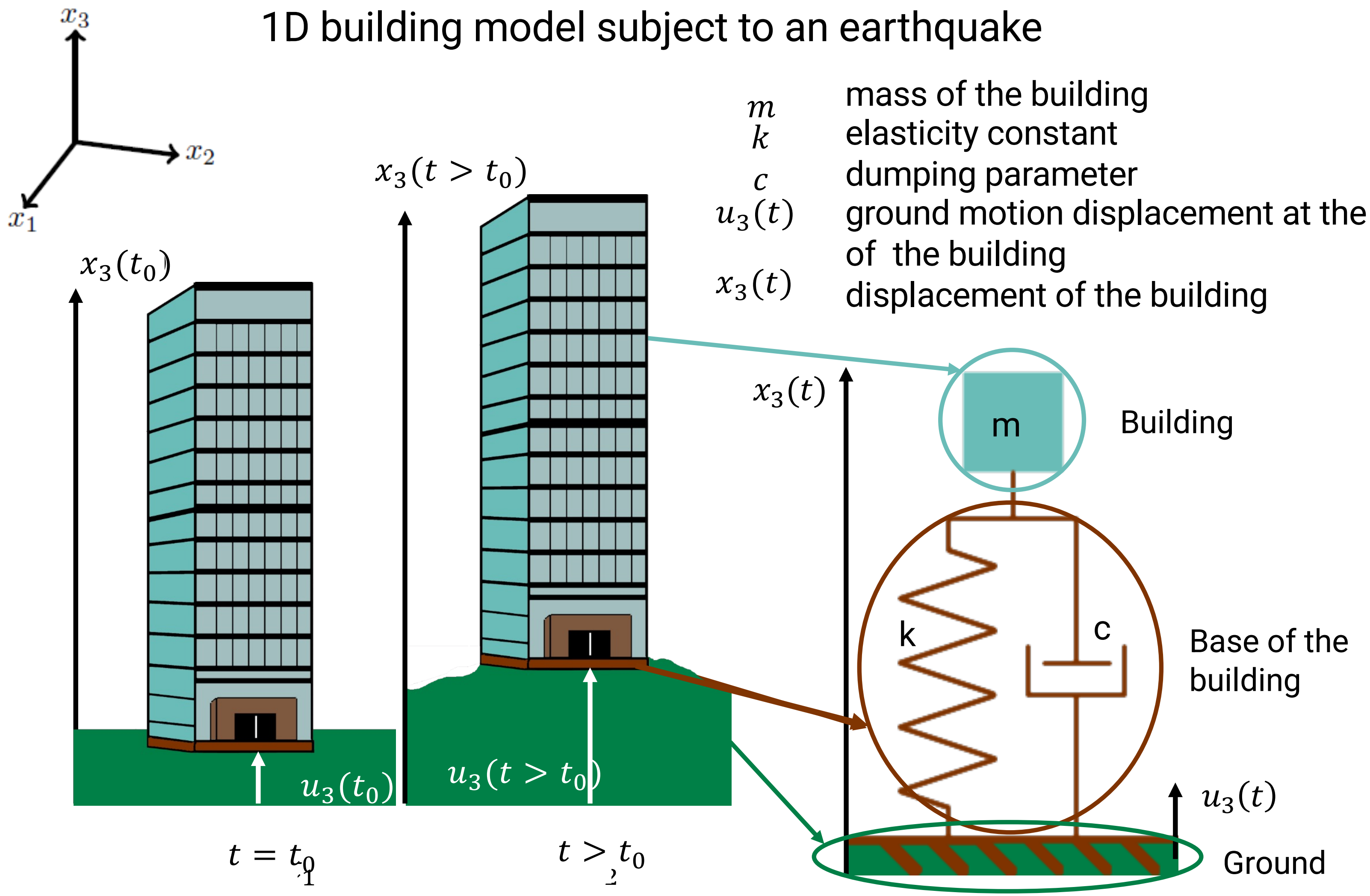
– Topography and 3D basin model
– Shunyi-Qianmen-Liangxiang fault
– Kinematic description of the seismic fault rupture]

[Gao, Yu, Zhang, Wu, Conference on Earthquake Engineering, 2004]
[Crempien, Archuleta, Seismol. Res. Lett., 2015]

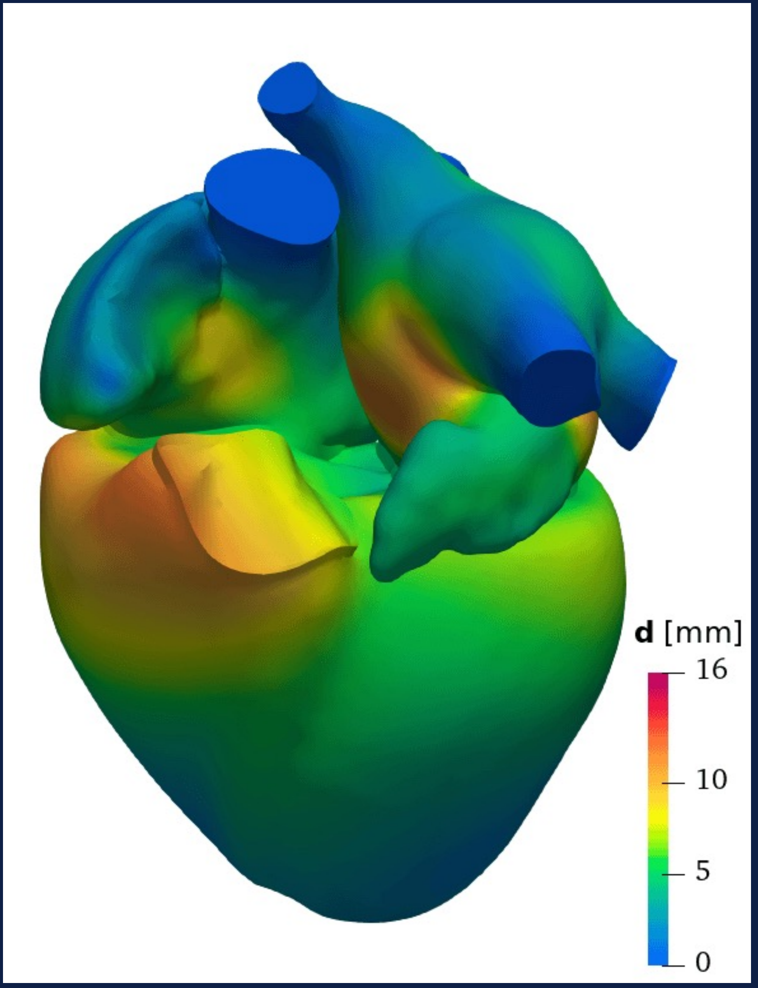
Ground motion parameters: spectral displacement

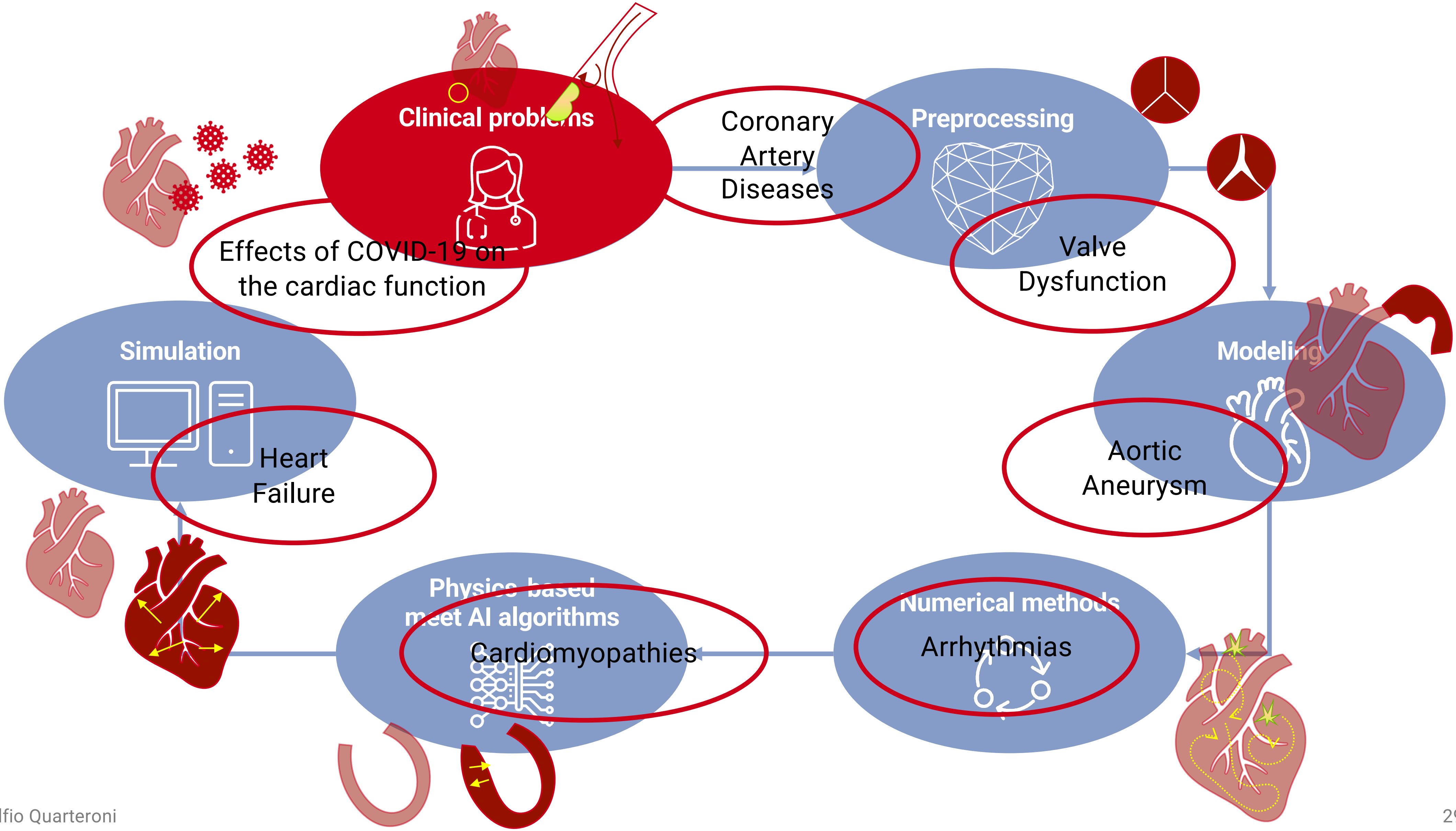
1D building model subject to an earthquake

m	mass of the building
k	elasticity constant
c	dumping parameter
$u_3(t)$	ground motion displacement at the base of the building
$x_3(t)$	displacement of the building



MATH for HEART




Clinical problems
Coronary
Artery
Diseases
Preprocessing
Valve
Dysfunction
Modeling
Aortic
Aneurysm
Numerical methods
Arrhythmias
Physics-based
meet AI algorithms
Cardiomyopathies
Simulation
Heart
Failure
Effects of COVID-19 on
the cardiac function

The Mathematical Heart

$$\begin{cases} \dfrac{\partial \mathbf{w}}{\partial t} = \mathbf{F}_{\mathrm{ion}}^{\mathbf{w}}(v, \mathbf{w}) & \text{in } \Omega \\ \dfrac{\partial \mathbf{z}}{\partial t} = \mathbf{F}_{\mathrm{ion}}^{\mathbf{z}}(v, \mathbf{w}, \mathbf{z}) & \text{in } \Omega \end{cases}$$

$$\begin{cases} J\chi C_{\mathrm{m}} \dfrac{\partial v}{\partial t} - \nabla \cdot (JF^{-1} D_{\mathrm{i}} F^{-T} \nabla (v + v_{\mathrm{e}})) \\ \qquad + J\chi I_{\mathrm{ion}}(v, \mathbf{w}, \mathbf{z}) = J\chi I_{\mathrm{app}}(\mathbf{x}, t) & \text{in } \Omega \\ -\nabla \cdot (JF^{-1} D_{\mathrm{i}} F^{-T} \nabla v) - \nabla \cdot (JF^{-1}(D_{\mathrm{i}} + D_{\mathrm{e}}) \nabla v_{\mathrm{e}}) = 0 & \text{in } \Omega \end{cases}$$

$$\mathbf{F}_{\mathrm{circ}}\left(\mathbf{c}, \frac{\mathrm{d}\mathbf{c}}{\mathrm{d}t}, t\right) = \mathbf{0}$$

Cellular Ions' dynamics

Cardiomyocites Contraction

Electrophysiology

External Circulation

$$\begin{cases} \dfrac{\partial \mathbf{s}}{\partial t} = \mathbf{F}_{\mathrm{act}}\left(\mathbf{s}, [\mathrm{Ca}^{2+}]_{\mathrm{i}}, \mathrm{SL}, \dfrac{\partial \mathrm{SL}}{\partial t}\right) & \text{in } \Omega \\ \mathrm{SL} = \mathrm{SL}_0 \sqrt{I_{4\mathrm{f}}} & \text{in } \Omega \end{cases}$$

$$P_{\mathrm{act}}(\mathbf{d}, \mathbf{s}) = T_{\mathrm{act}}\left(n_{\mathrm{f}} \frac{F\mathbf{f}_0 \cdot \mathbf{f}_0}{\sqrt{I_{4\mathrm{f}}}} + n_{\mathrm{s}} \frac{F\mathbf{s}_0 \cdot \mathbf{s}_0}{\sqrt{I_{4\mathrm{s}}}} + n_{\mathrm{n}} \frac{F\mathbf{n}_0 \cdot \mathbf{n}_0}{\sqrt{I_{4\mathrm{n}}}}\right)$$

$$T_{\mathrm{act}} = T_{\mathrm{act}}(\mathbf{s}, \mathrm{SL}), \qquad I_{4\mathrm{i}} = F\mathbf{i}_0 \cdot F\mathbf{i}_0 \quad \mathrm{i} \in \{\mathbf{f}, \mathbf{s}, \mathbf{n}\}$$

$$\rho_{\mathrm{f}}\left[\frac{\partial \mathbf{u}}{\partial t} + ((\mathbf{u} - \mathbf{u}_{\mathrm{ALE}}) \cdot \nabla)\, \mathbf{u}\right] - \nabla \cdot \sigma_{\mathrm{f}}(\mathbf{u}, p) + \boldsymbol{\mathcal{R}}(\mathbf{u}, \mathbf{u}_{\mathrm{ALE}}) = \mathbf{0}$$

$$\boldsymbol{\mathcal{R}}(\mathbf{u}, \mathbf{u}_{\mathrm{ALE}}) = \sum_{\mathrm{k} \in \mathcal{V}} \frac{R_{\mathrm{k}}}{\varepsilon_{\mathrm{k}}} \delta_{\varepsilon_{\mathrm{k}}}\left(\varphi_{\mathrm{k}}^{t}(\mathbf{x})\right)(\mathbf{u} - \mathbf{u}_{\mathrm{ALE}} - \mathbf{u}_{\Gamma_{\mathrm{k}}})$$

Valves Dynamics

$$\begin{cases} \mathrm{NS}(\mathbf{u}, p) = \mathbf{0} & \text{in } \Omega_{\mathrm{cor}} \\ \mathbf{K}_i^{-1} \mathbf{u}_{\mathrm{myo},i} + \boldsymbol{\nabla} p_{\mathrm{myo},i} = \mathbf{0}, \quad i = 1, 2, 3 & \text{in } \Omega_{\mathrm{myo}} \\ \boldsymbol{\nabla} \cdot \mathbf{u}_{\mathrm{myo},1} = \displaystyle\sum_{j=1}^{J} \frac{\chi_{\Omega_{\mathrm{myo}}^{j}}}{|\Omega_{\mathrm{myo}}^{j}|} \int_{\Gamma_j^{\mathrm{coro}}} \mathbf{u} \cdot \mathbf{n} - \beta_{1,2}(p_{\mathrm{myo},1} - p_{\mathrm{myo},2}) & \text{in } \Omega_{\mathrm{myo}} \\ \boldsymbol{\nabla} \cdot \mathbf{u}_{\mathrm{myo},2} = -\beta_{2,1}(p_{\mathrm{myo},2} - p_{\mathrm{myo},1}) - \beta_{2,3}(p_{\mathrm{myo},2} - p_{\mathrm{myo},3}) & \text{in } \Omega_{\mathrm{myo}} \\ \boldsymbol{\nabla} \cdot \mathbf{u}_{\mathrm{myo},3} = -\gamma(p_{\mathrm{myo},3} - p_{\mathrm{veins}}) - \beta_{3,2}(p_{\mathrm{myo},3} - p_{\mathrm{myo},2}) & \text{in } \Omega_{\mathrm{myo}} \end{cases}$$

Myocardial Perfusion

$$\begin{cases} -\nabla \cdot P_{\mathrm{ALE}}(\mathbf{d}_{\mathrm{ALE}}) = \mathbf{0} & \text{in } \widehat{\Omega} \\ \mathbf{d}_{\mathrm{ALE}} = \mathbf{d} & \text{on } \hat{\Sigma} \end{cases} \qquad \mathbf{u}_{\mathrm{ALE}} = \frac{\partial \mathbf{d}_{\mathrm{ALE}}}{\partial t}$$

$$\begin{cases} \rho_{\mathrm{f}}\left[\dfrac{\partial \mathbf{u}}{\partial t} + ((\mathbf{u} - \mathbf{u}_{\mathrm{ALE}}) \cdot \nabla)\, \mathbf{u}\right] - \nabla \cdot \sigma_{\mathrm{f}}(\mathbf{u}, p) = \mathbf{0} & \text{in } \Omega \\ \nabla \cdot \mathbf{u} = 0 & \text{in } \Omega \end{cases}$$

$$\sigma_{\mathrm{f}}(\mathbf{u}, p) = \mu\left(\nabla \mathbf{u} + \nabla \mathbf{u}^{T}\right) - pI$$

Blood Dynamics, Contraction & Relaxation

ECG Reconstruction

$$\rho_{\mathrm{s}} \frac{\partial^2 \mathbf{d}}{\partial t^2} - \nabla \cdot P_{\mathrm{s}}(\mathbf{d}, \mathbf{s}) = \mathbf{0} \qquad \text{in } \Omega$$

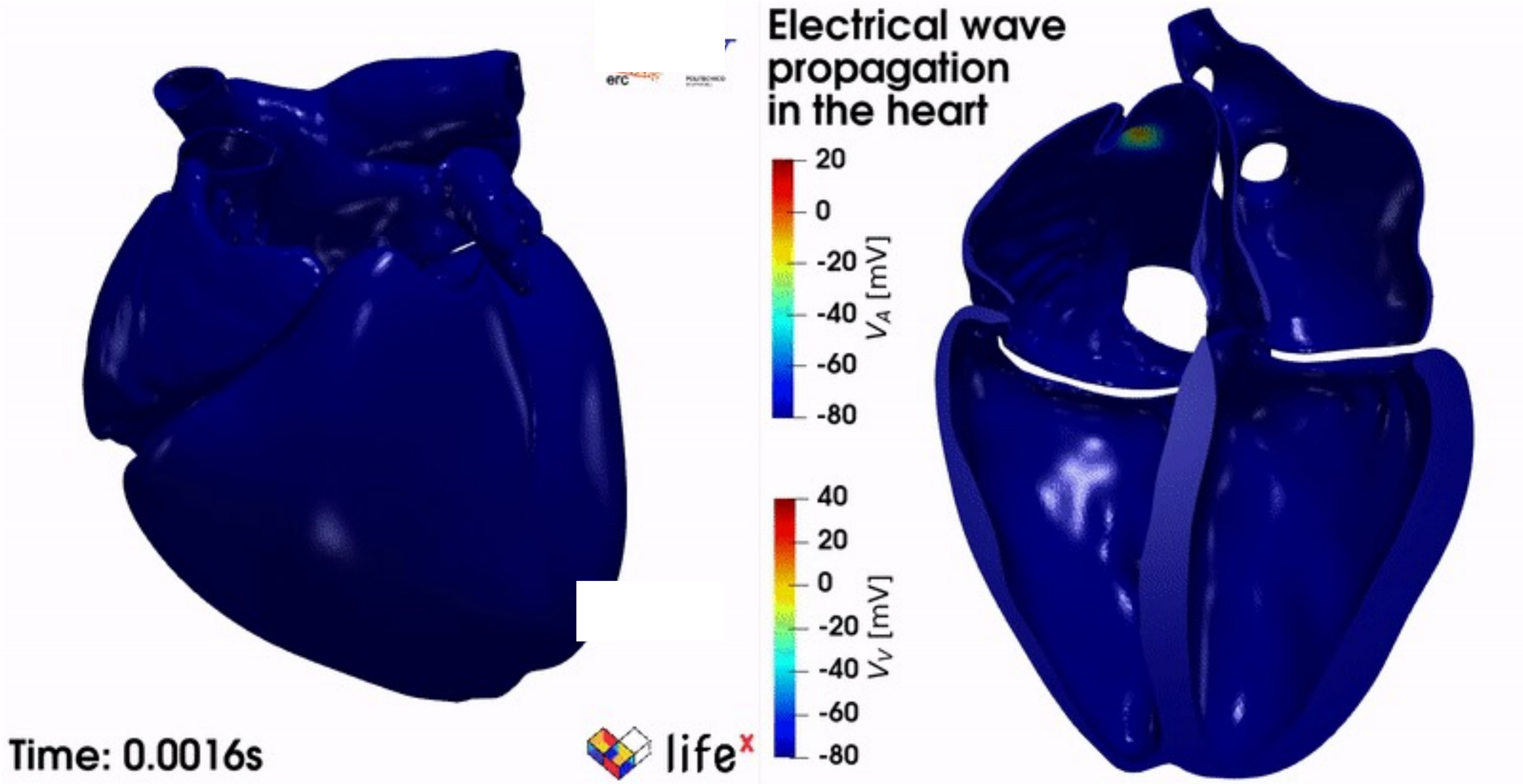
$$P_{\mathrm{s}}(\mathbf{d}, \mathbf{s}) = P_{\mathrm{pas}}(\mathbf{d}) + P_{\mathrm{act}}(\mathbf{d}, \mathbf{s})$$

$$P_{\mathrm{pas}}(\mathbf{d}) = \frac{\partial \mathcal{W}}{\partial F} \qquad F = I + \nabla \mathbf{d}$$

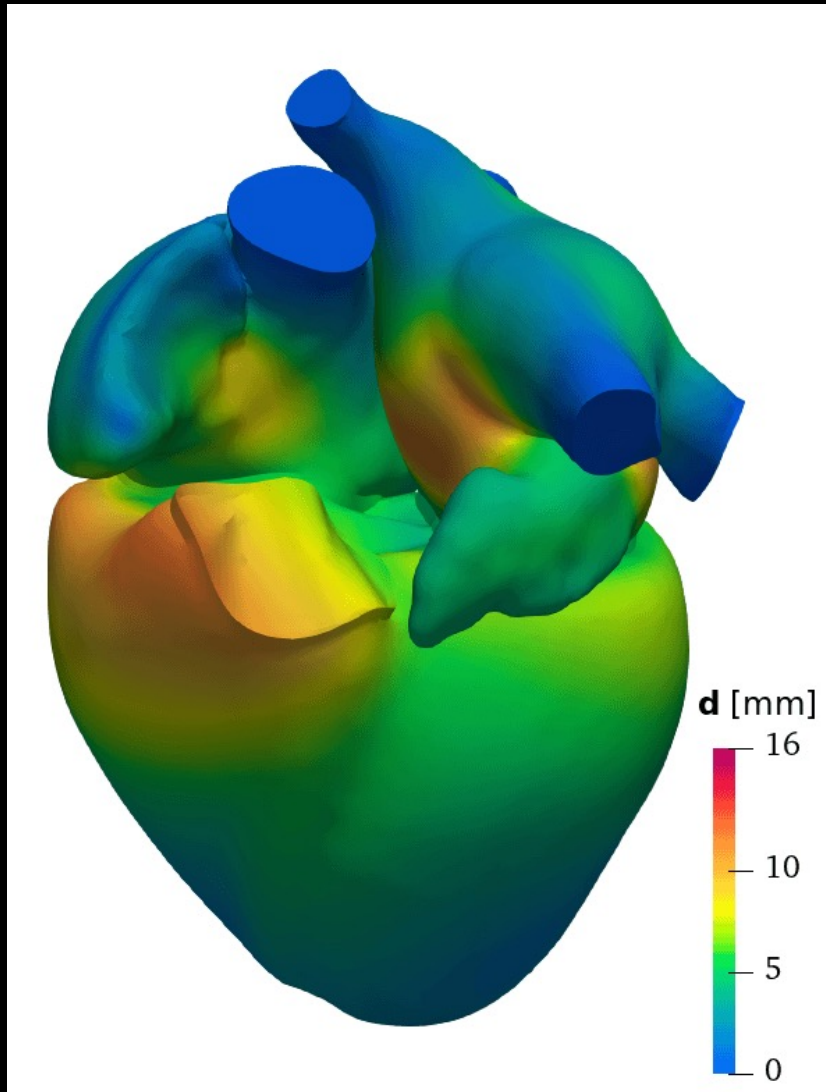
$$\begin{cases} \mathrm{EP}(v, v_e) = 0 & \text{in } \Omega_H \\ -\nabla \cdot (D_T \nabla v_T) = 0 & \text{in } \Omega_{\mathrm{torso}} \\ v_T = v_e & \text{on } \Sigma \\ D_T \nabla v_T \cdot \mathbf{n}_H = D_e \nabla v_e \cdot \mathbf{n}_H & \text{on } \Sigma \end{cases}$$

Whole Heart electrophysiology simulation

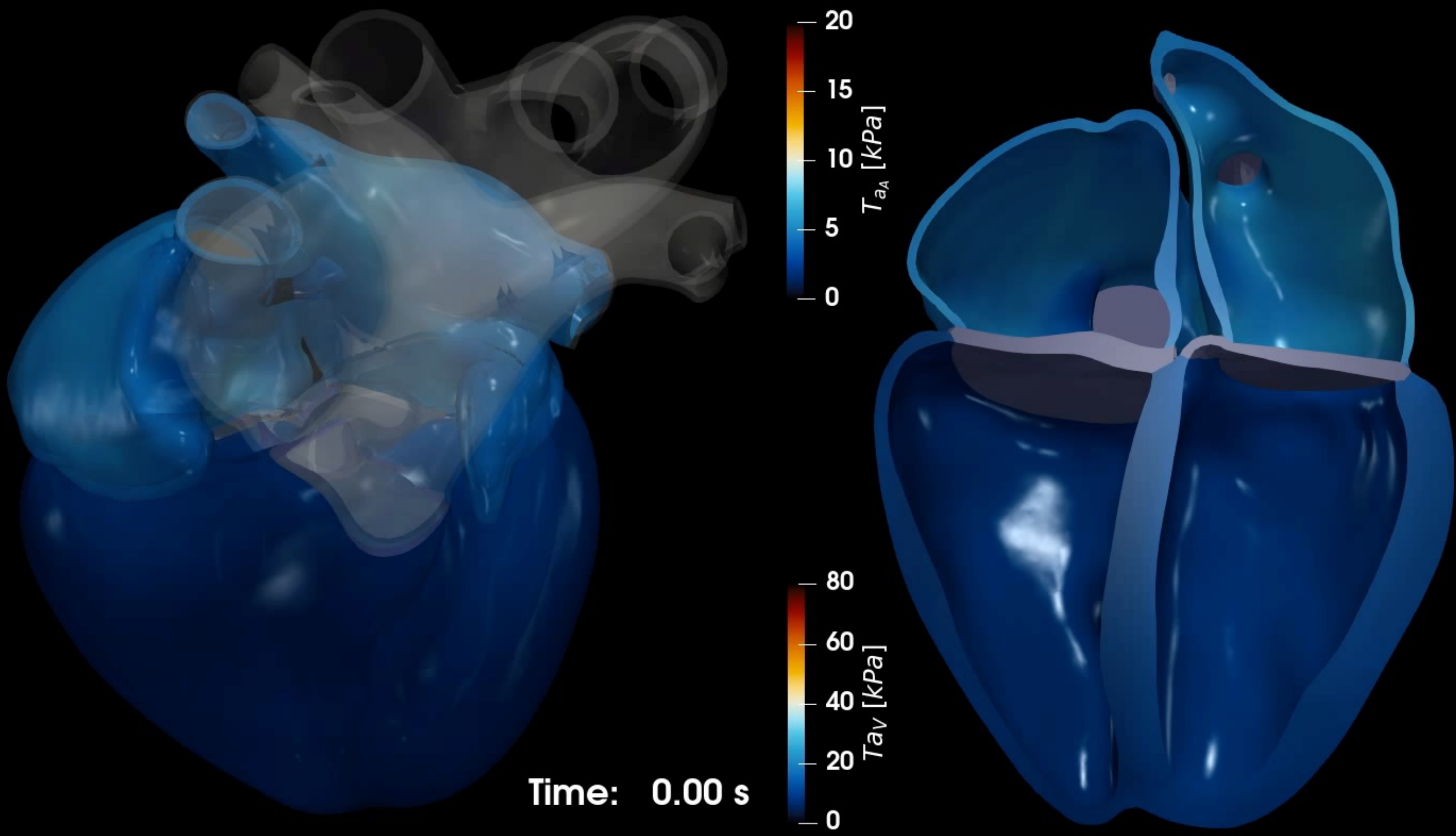
Time evolution of the ventricular (V_V) and atrial (V_A) transmembrane potential

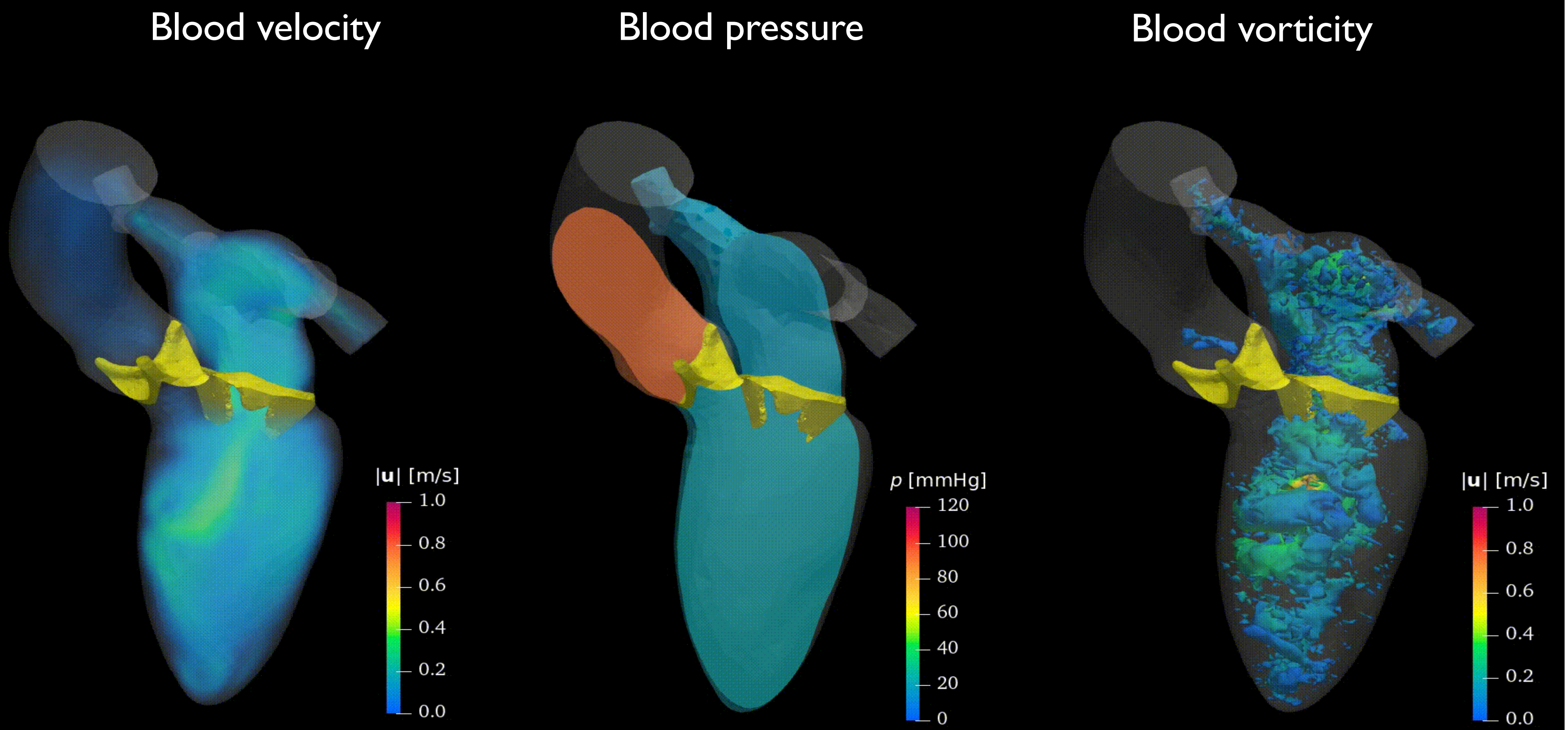


Displacement (Contraction and Relaxation)

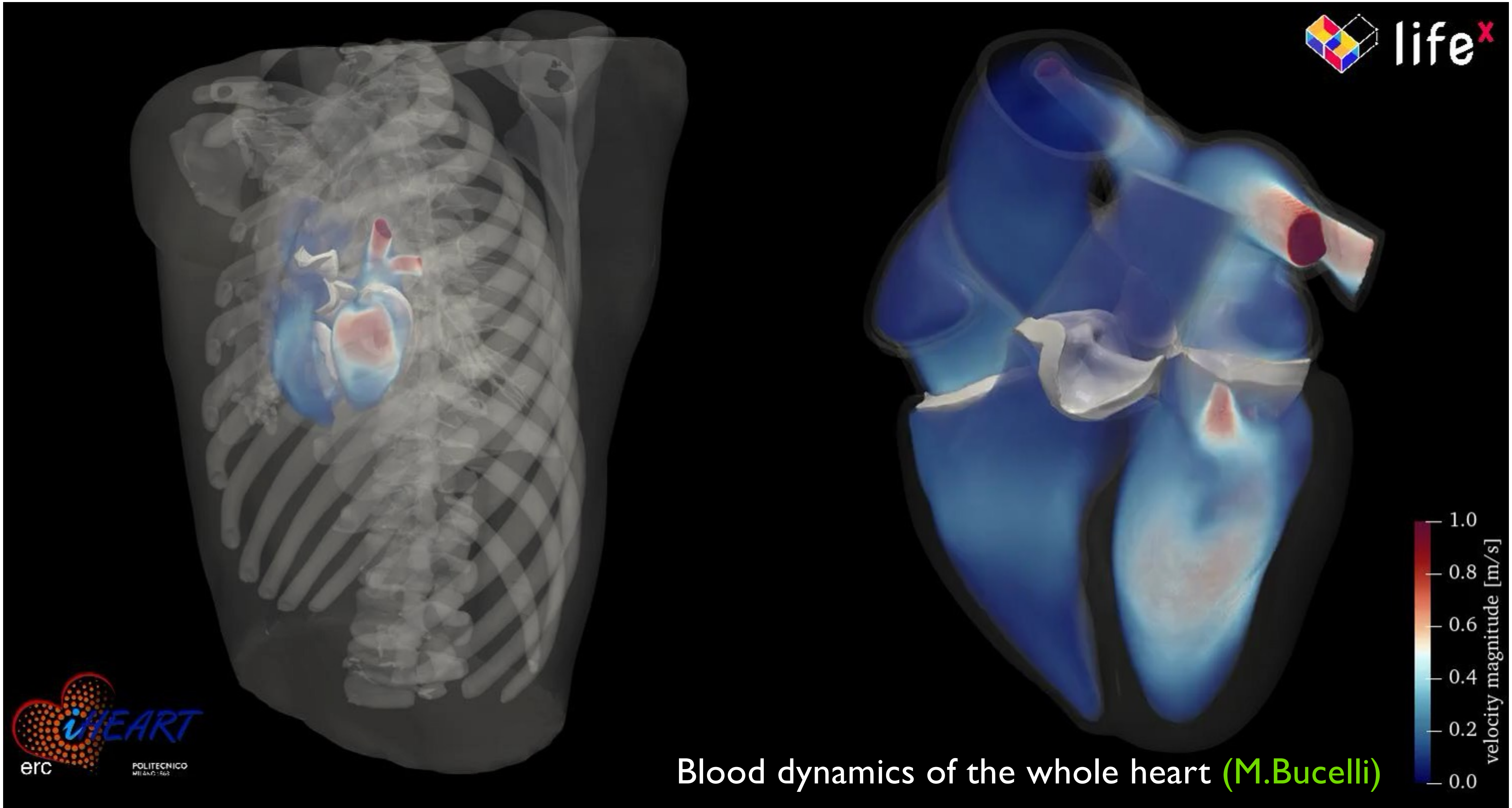


Active Tension at Cardiomyocites



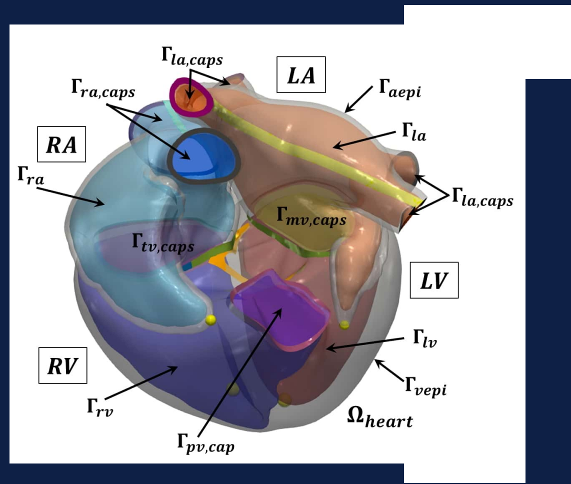


Fluid dynamics of the left heart (A.Zingaro)



Blood dynamics of the whole heart (M.Bucelli)

A Few Selected Clinical Applications



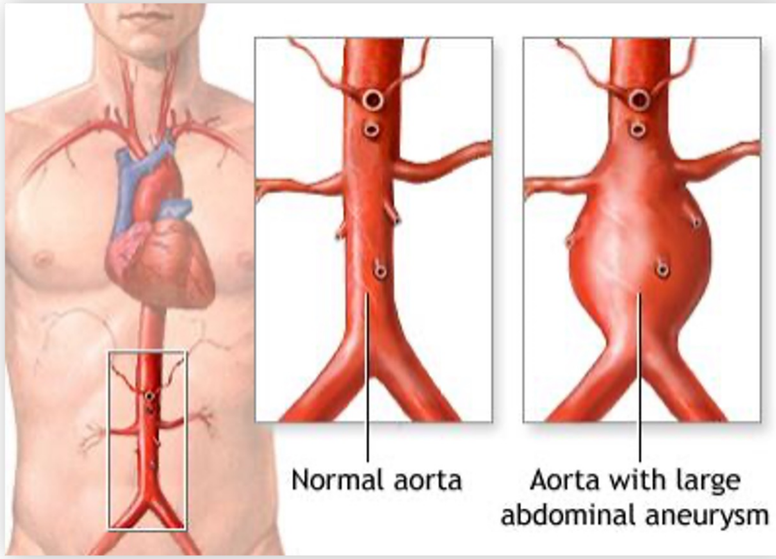
Risk assessment for AAA

(Abdominal Aortic Aneurisms)

Risk assessment of abdominal aortic aneurysm (AAA)

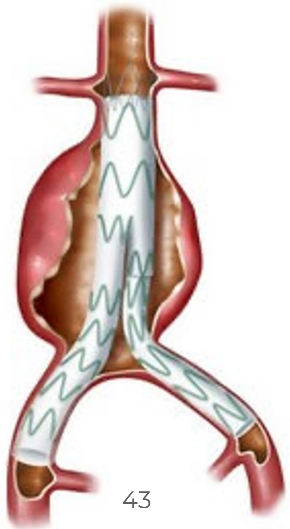
Problem:

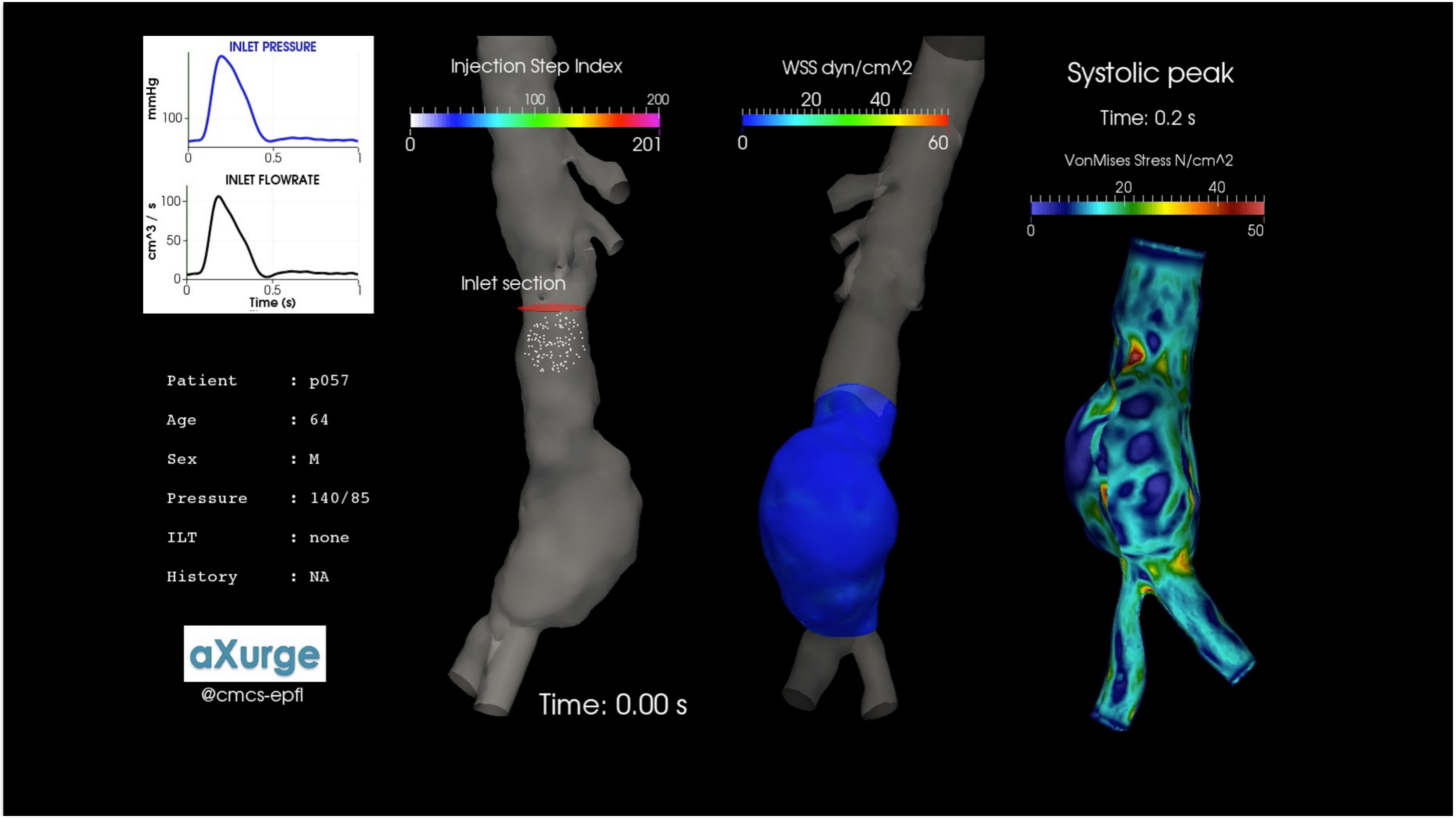
- Affects 8% of males over the age of 65
- Mortality if ruptured is 85-90%
- Surgical intervention if AAA diameter > 5 cm
- Diameter manually measured
- Proven not to be the best predictive choice



Methods:

- Semi-automatic CT segmentation
- Mesh generation and fluid/mechanical simulations

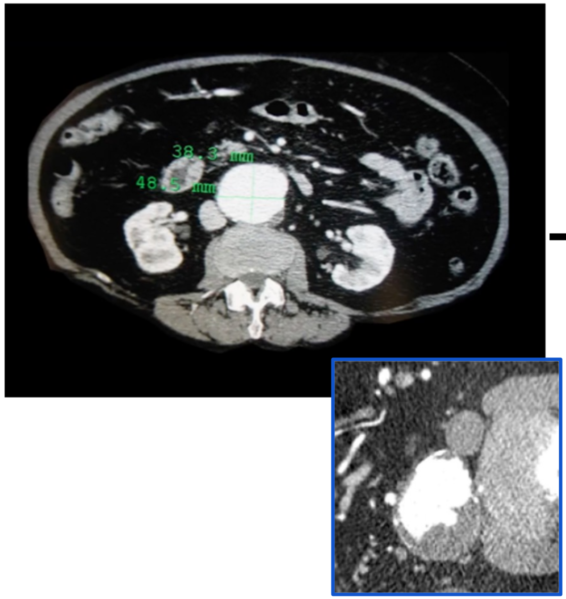




INLET PRESSURE
mmHg
100
0
0.5
1
INLET FLOWRATE
cm^3 / s
100
50
0
0
0.5
1
Time (s)
Injection Step Index
100
200
0
201
Inlet section
WSS dyn/cm^2
20
40
0
60
Systolic peak
Time: 0.2 s
VonMises Stress N/cm^2
20
40
0
50
Patient : p057
Age : 64
Sex : M
Pressure : 140/85
ILT : none
History : NA
aXurge
@cmcs-epfl
Time: 0.00 s

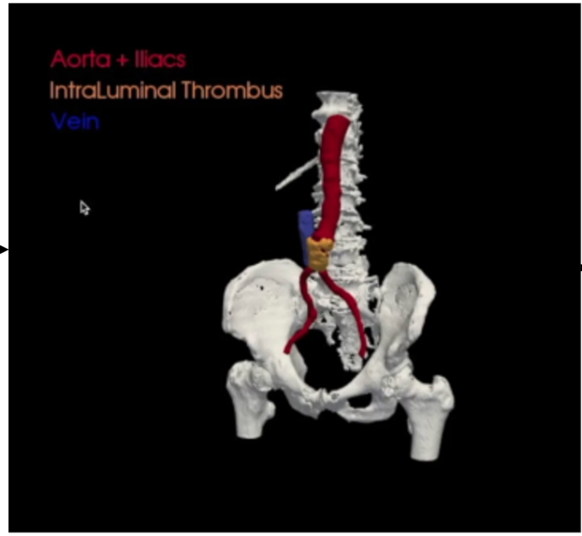
Risk assessment of abdominal aortic aneurysm (AAA)

CT scan

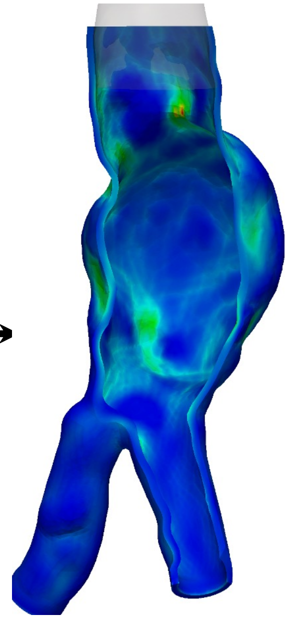


Segment

Surface/Mesh

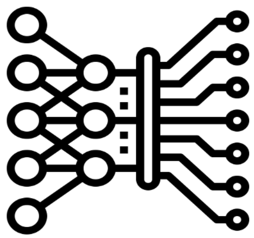


Numerical simulations

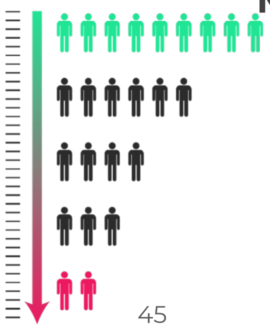


Fluid / Mechanical measures

MACHINE LEARNING



Low risk



High risk

RISK ASSESSMENT

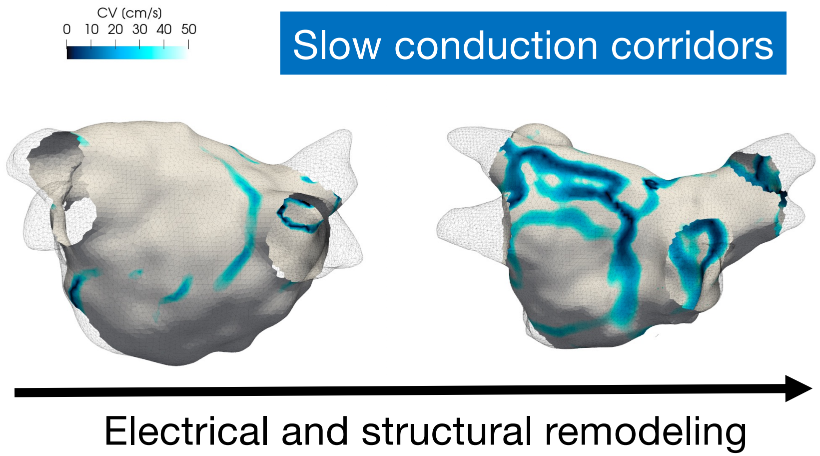
Atrial fibrillation (AF)

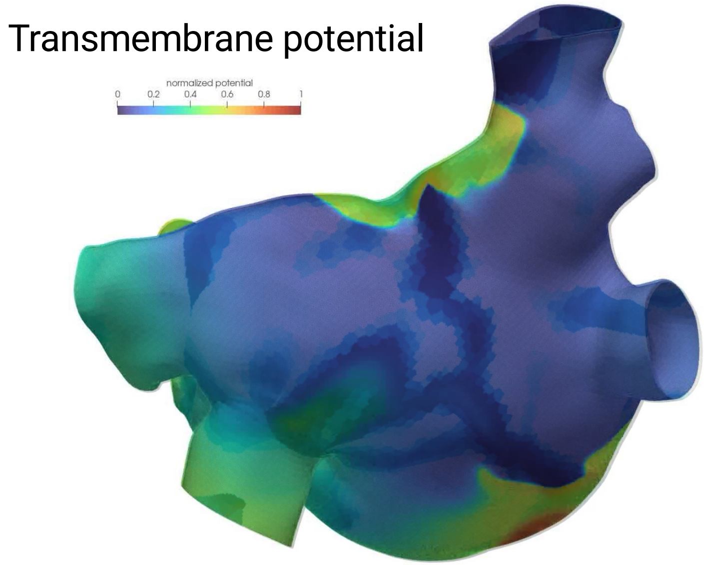
Simulation

Clinical question: which are the mechanisms behind AF progression?

IRCCS HUMANITAS RESEARCH HOSPITAL

I.R.C.C.S. Ospedale San Raffaele





- **slow conduction corridors** and **pivot points** quantitatively characterize **AF progression**
- Numerical simulations confirm the role of **slow conduction corridors** in **AF sustainment** (localized reentry anchoring)

A. Frontera, S. Pagani, L.R. Limite et al., *JACC: Clinical Electrophysiology*, 2022
S. Pagani, L. Dede', A. Frontera et al., *Frontiers in Physiology*, 2021

Hypertrophic Cardiomyopathy (HCM)

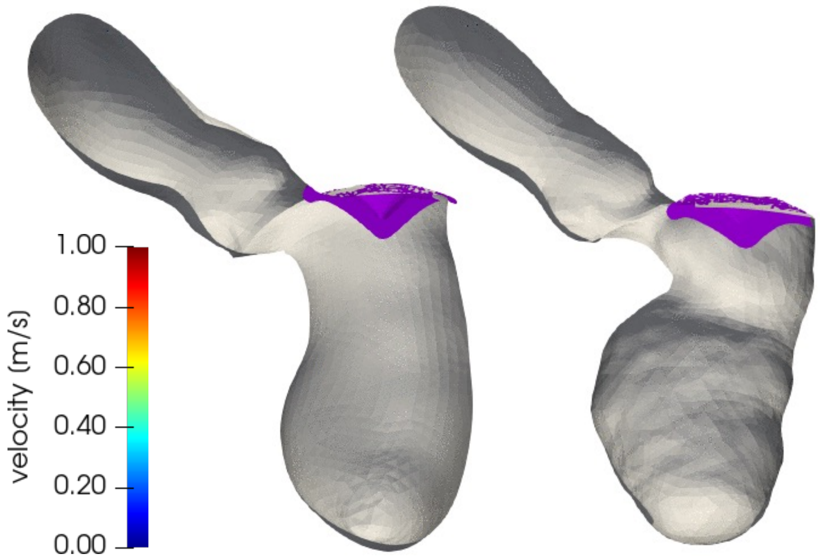
Clinical question: can CFD simulations guide obstruction assessment and pre-operative design of septal myectomy?

Ospedale Luigi Sacco
AZIENDA OSPEDALIERA - POLO UNIVERSITARIO

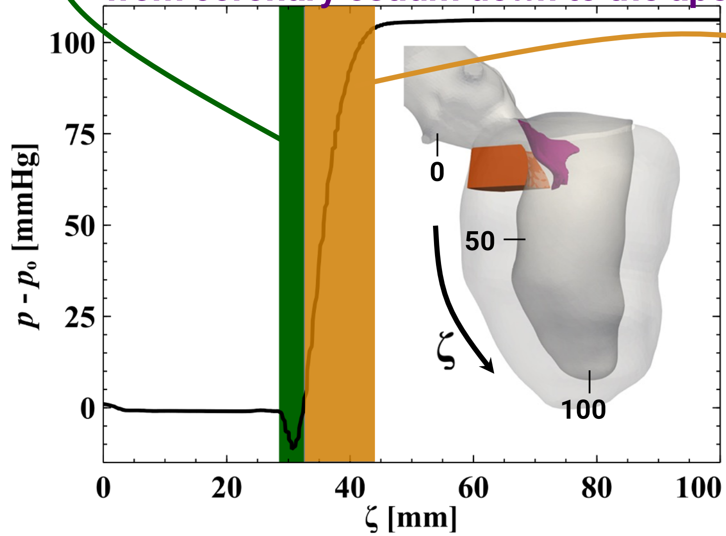
SISTEMA SANITARIO REGIONALE
AZIENDA OSPEDALIERA SAN CAMILLO FORLANINI

Obstructive HCM-induced **Venturi effect** causing systolic anterior motion

non-obstructive HCM **obstructive HCM**



pressure along septum at systolic peak from coronary ostium down to the apex



obstruction severity: **extent** and **amplitude** of **subaortic pressure gradient**

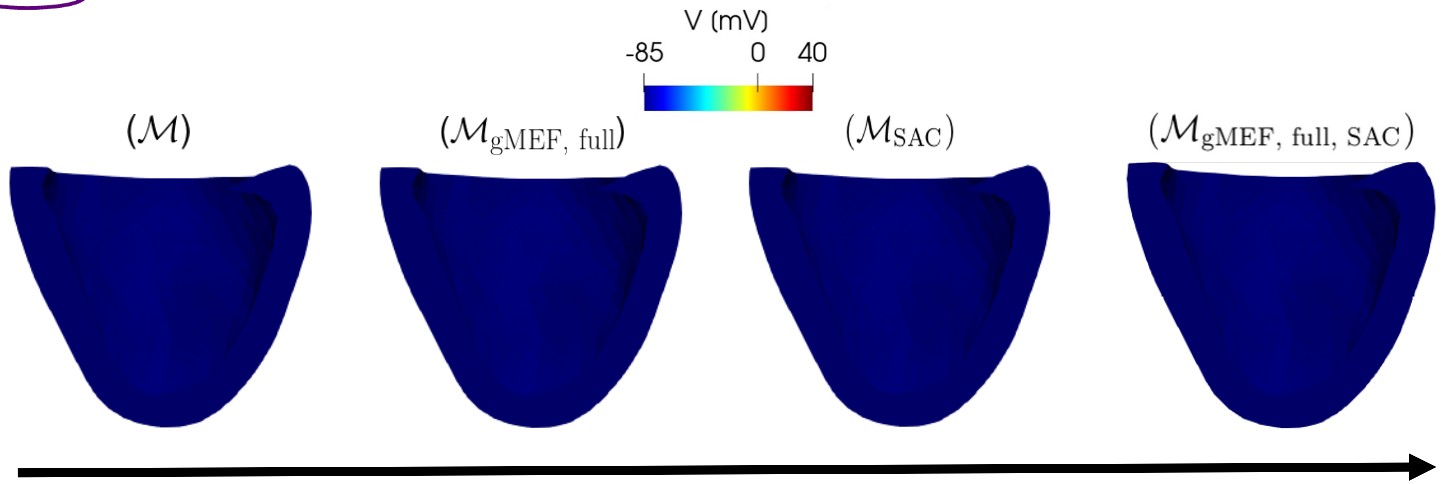
indication for surgical treatment: **portion to remove** by septal myectomy

I. Fumagalli, M. Fedele, C. Vergara, et al., *Computers in Biology and Medicine*, 2020
I. Fumagalli, P. Vitullo, C. Vergara, et al., *Frontiers in Physiology* , 2022

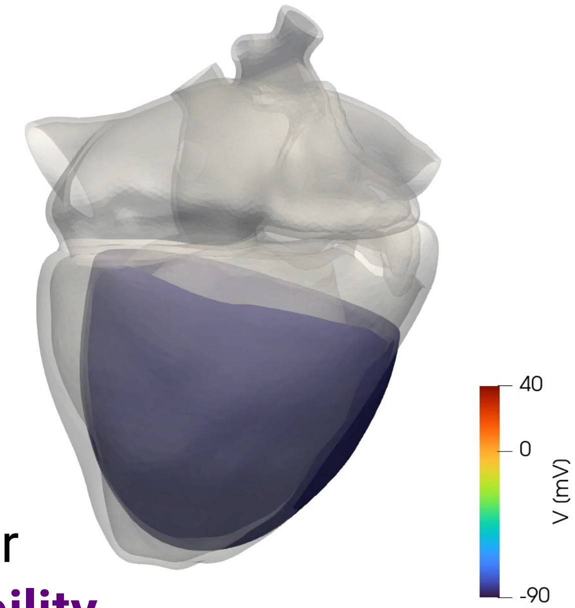
Ventricular tachycardia and fibrillation

Clinical question: are ventricular tachycardia and fibrillation better simulated by accounting for mechanical deformation?

JOHNS HOPKINS UNIVERSITY



From **electrophysiology** to **electromechanics**
(**geometry-mediated mechano-electric feedbacks** and **stretch-activated channels**)



Electrophysiology and **electromechanics** simulations may differ in **conduction velocity**, **electric stability** and **hemodynamic stability**

M. Salvador, M. Fedele, P.C. Africa et al., *Computers in Biology and Medicine*, 2021
M. Salvador, F. Regazzoni, S. Pagani et al., *Computers in Biology and Medicine*, 2022

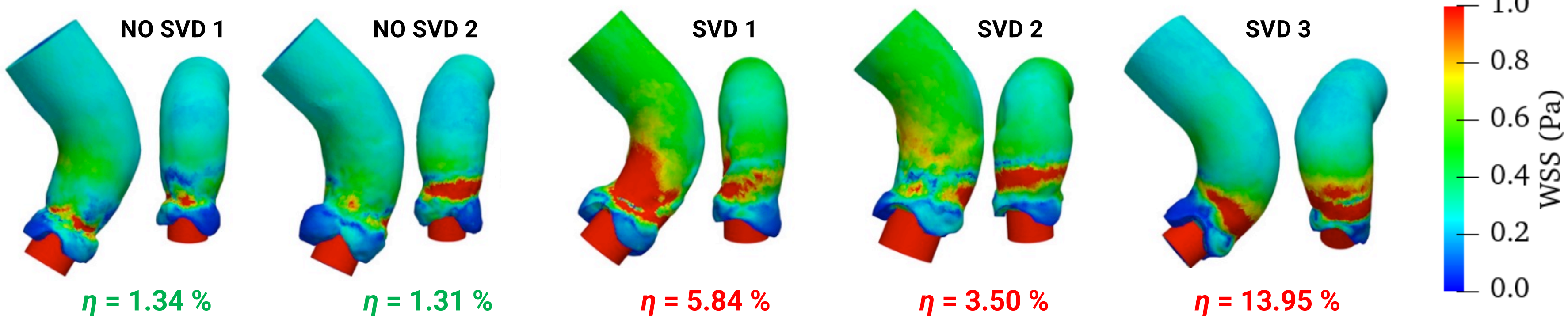
Simulation

Transcatheter Aortic Valve Implantation (TAVI)

Clinical question: which are the predictive indicators of TAVI Structural Valve Deterioration (SVD)?

Centro Cardiologico Monzino

WSS at systolic peak



- Analysis based on **pre-implantation** data only
- **WSS stronger** and more **persistent** in **SVD** cases
- ***η* index** discriminating SVD from NO-SVD, based on **Time-Averaged WSS (TAWSS) Critical Area (CA)**:

$$\eta = \frac{|CA|}{|\Gamma_{\text{wall}}|}, \text{ with } CA = \{\mathbf{x} \in \Gamma_{\text{wall}} : TAWSS(\mathbf{x}) > 0.5\text{Pa}\}$$

I. Fumagalli, R. Polidori, F. Renzi et al., *MOX Report,* Politecnico di Milano, 2022

Estimating cardiac blood flow maps

Clinical question: can we replace CT scans and stress protocols with a computational estimation of myocardial blood flow maps?

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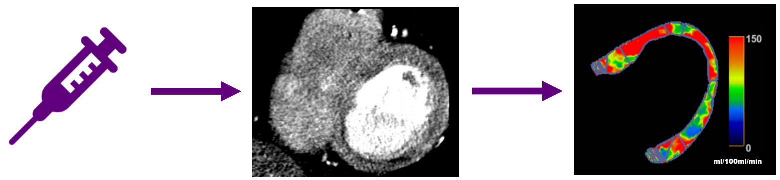
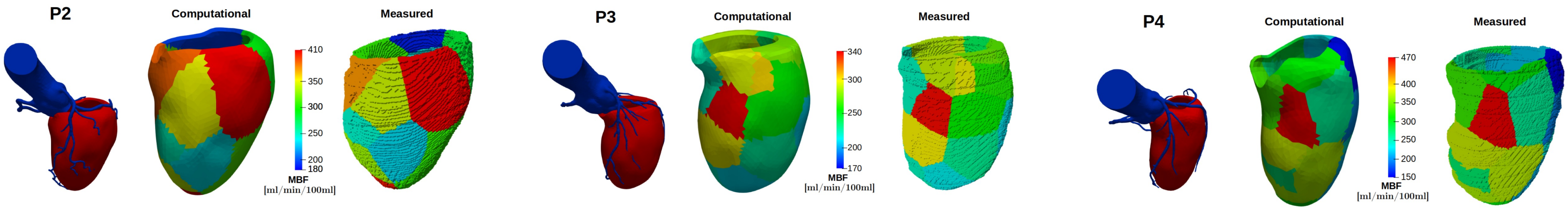
Clinical pipeline



Consistency tests: calibration of **perfusion model** on available maps yields **excellent agreement**

Ongoing: calibration of patient-specific models based on **pressure data only** (no maps)



S. Di Gregorio, C. Vergara, G. Montino Pelagi et al., *European Journal of Nuclear Medicine and Molecular Imaging* 2022

Dealing with Complexity

How Fast Supercomputers Are?

FRONTIER: 1.6 Billion Billion FLOPS

8 Billion People on Earth

200 Million Operations per Second per Inhabitant

A complete simulation of a single heartbeat (I

Requires at least 20M degrees of freedom on 192 cores on a supercomputing facility

May last up to 48 hours

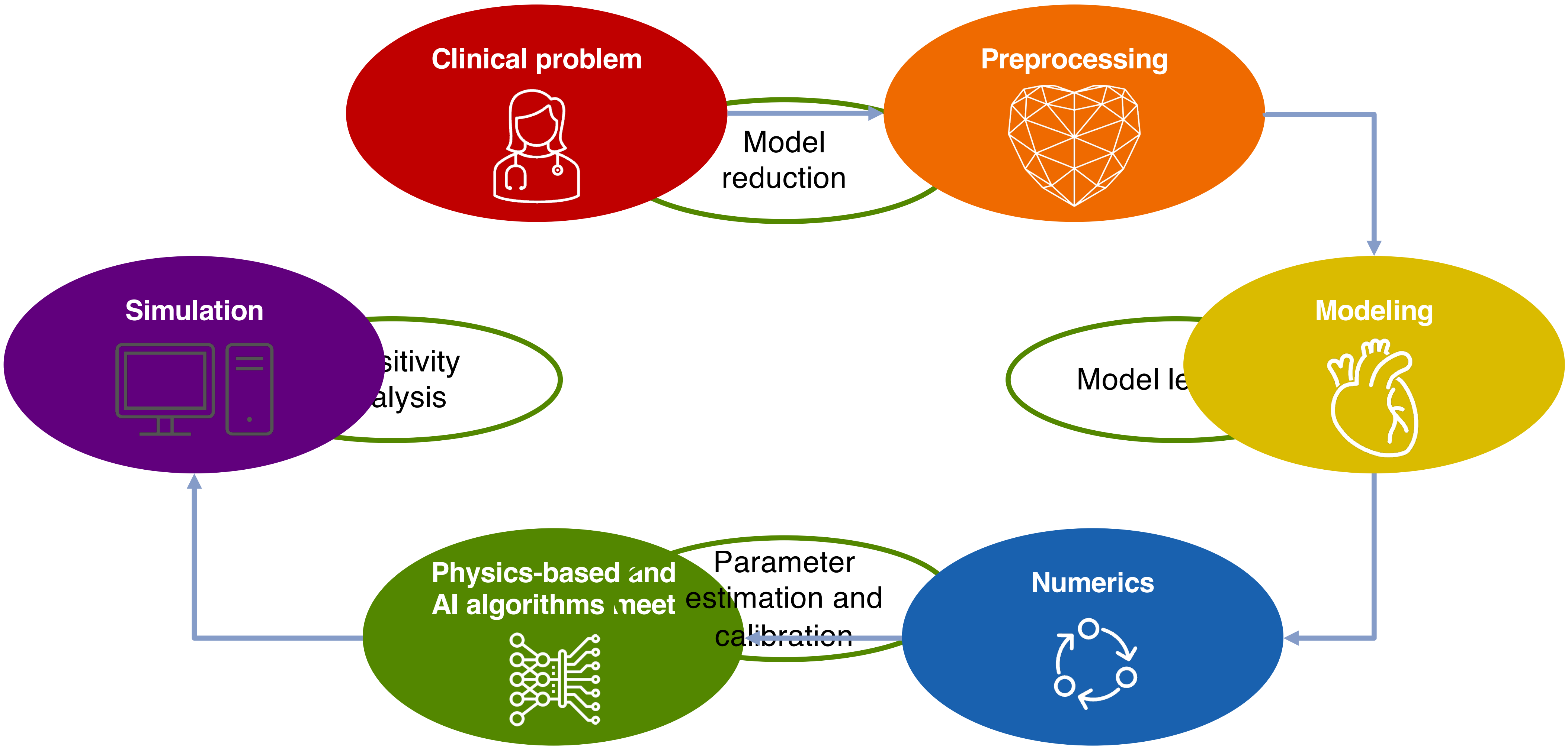
Costs about 2000 euros

Consumes 100kWh of energy

Produces 35kg of CO_2 (without accounting for the additional CO_2 produced for the cooling of the cluster)

Developing better models and more efficient and accurate numerical methods is of paramount importance

A new exi
in Enginee
Rapid and r


Clinical problem
Model
reduction
Preprocessing
Simulation
sitivity
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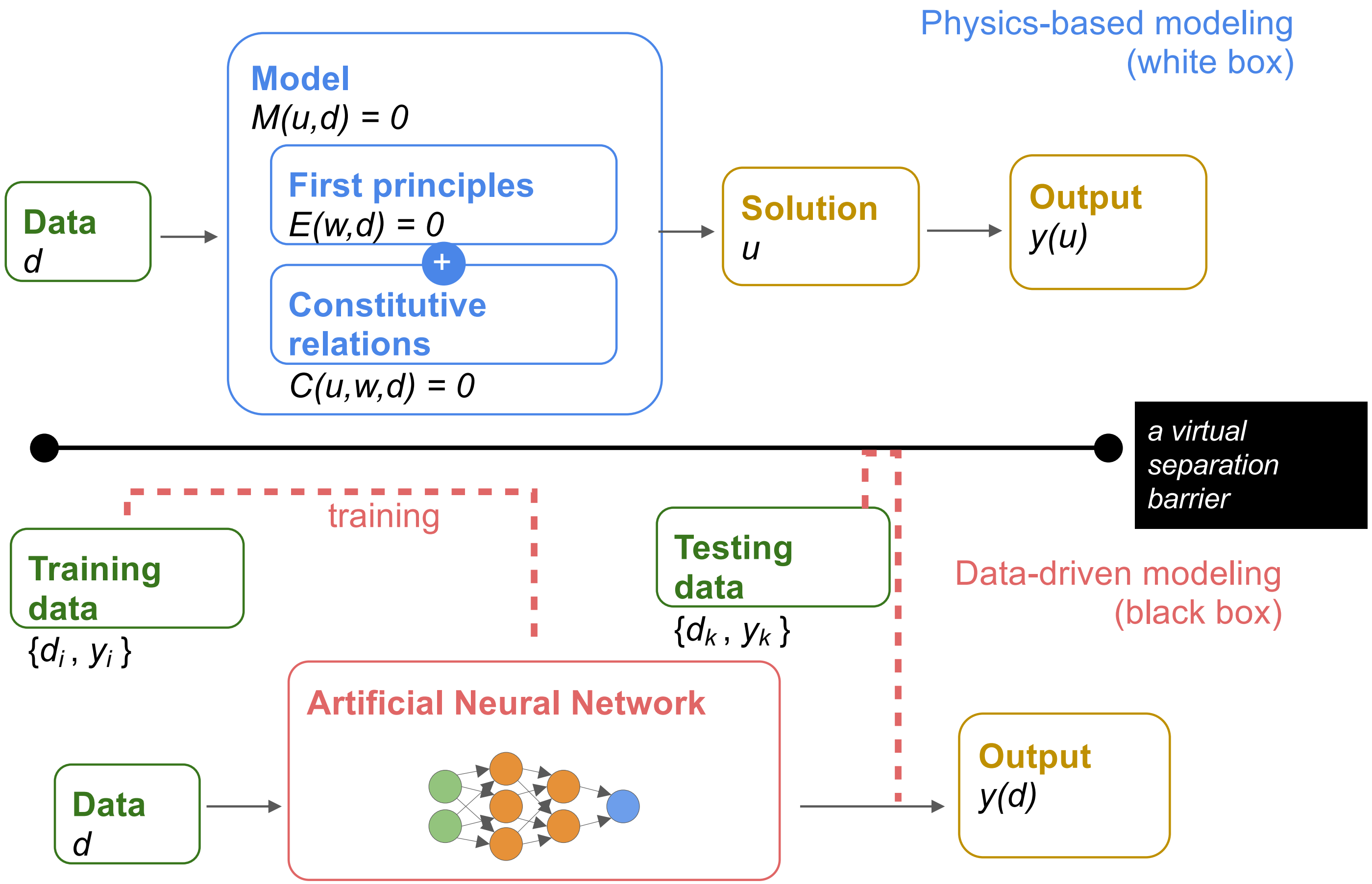
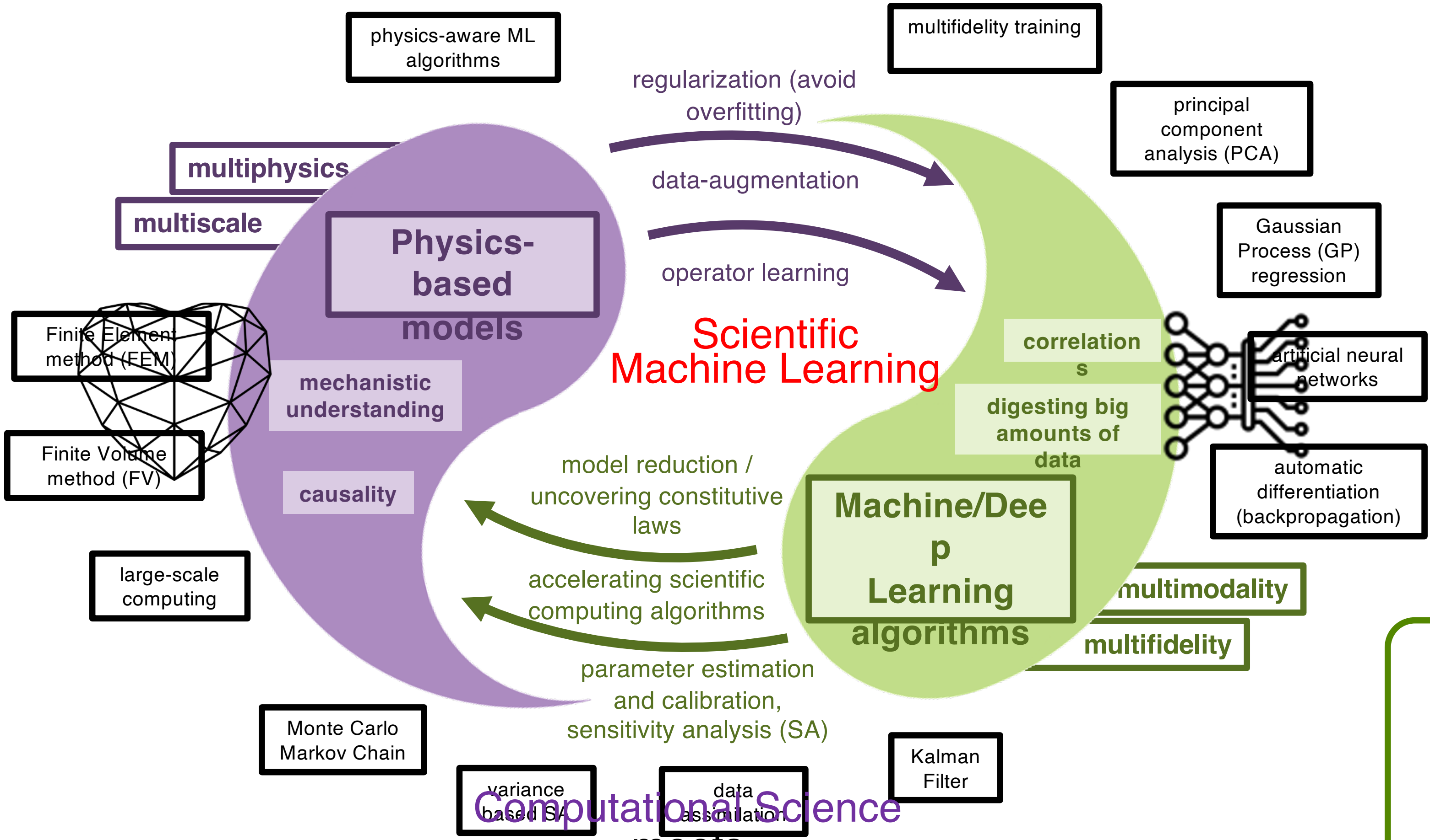
Model le
Modeling
Physics-based and
AI algorithms meet
Parameter
estimation and
calibration
Numerics

Physics-based vs Data-driven modeling

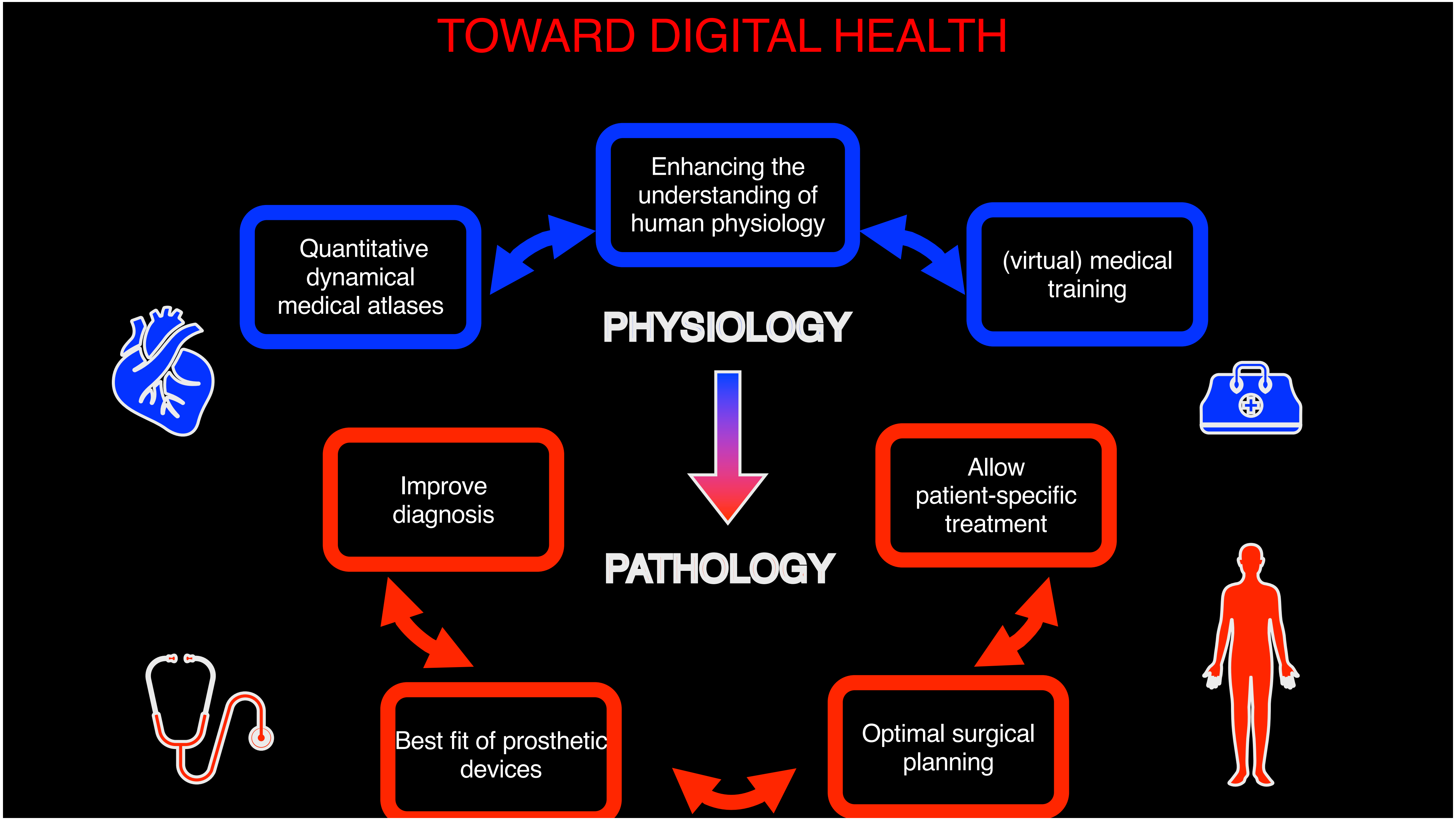


A cooperative game

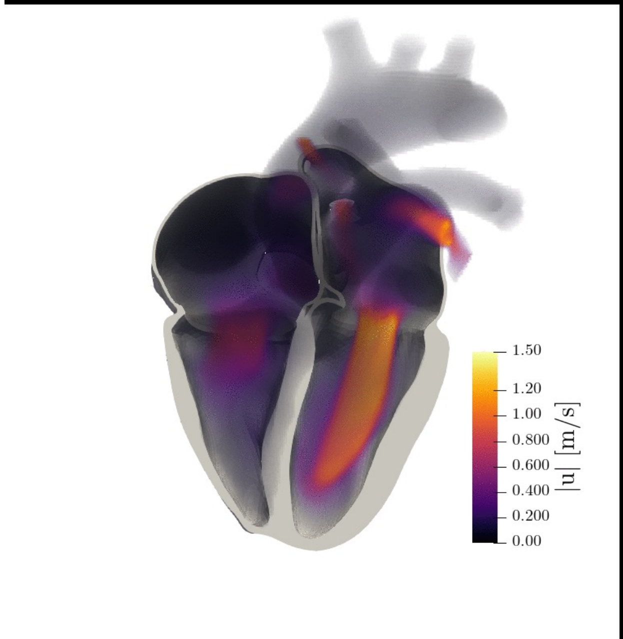


Computational Science
meets
Computational Learning

scan for info & references


TOWARD DIGITAL HEALTH
Enhancing the understanding of human physiology
Quantitative dynamical medical atlases
(virtual) medical training
PHYSIOLOGY
PATHOLOGY
Improve diagnosis
Allow patient-specific treatment
Best fit of prosthetic devices
Optimal surgical planning

THANK YOU



1.50
1.20
1.00
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0.600
0.400
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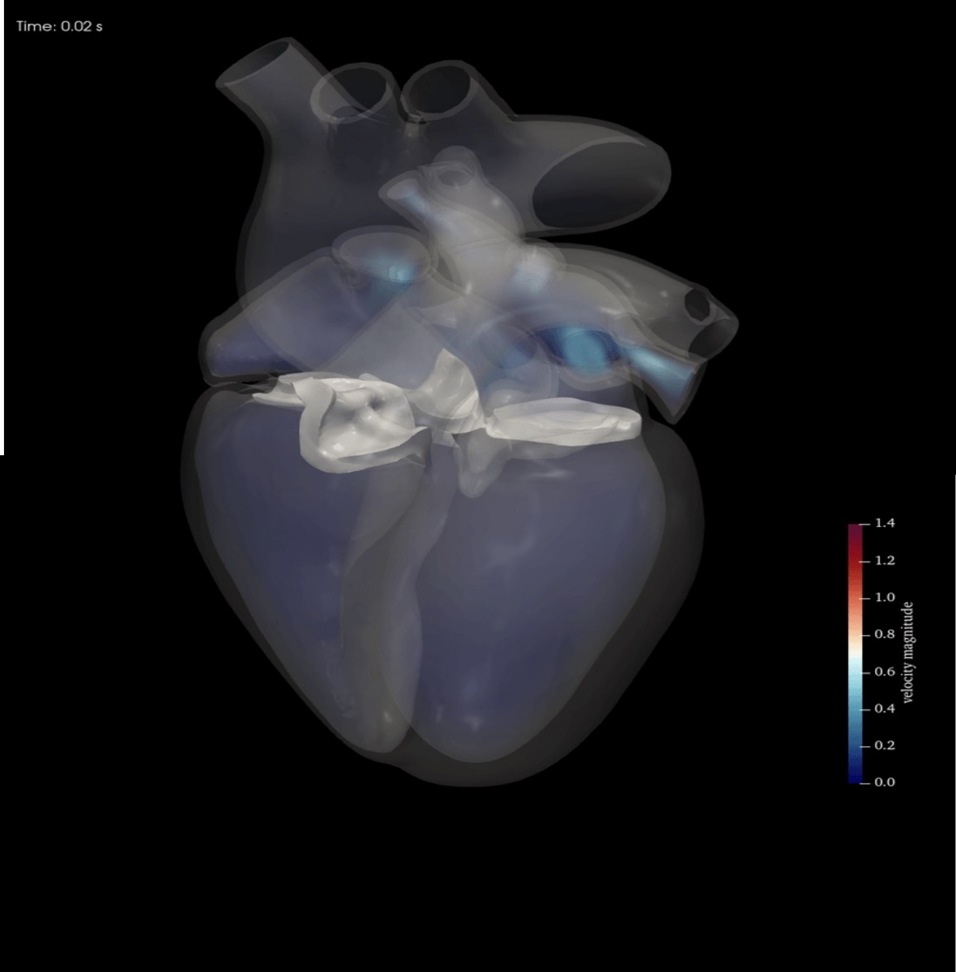
0.00
|u| [m/s]



Time: 0.02 s
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1.2
1.0
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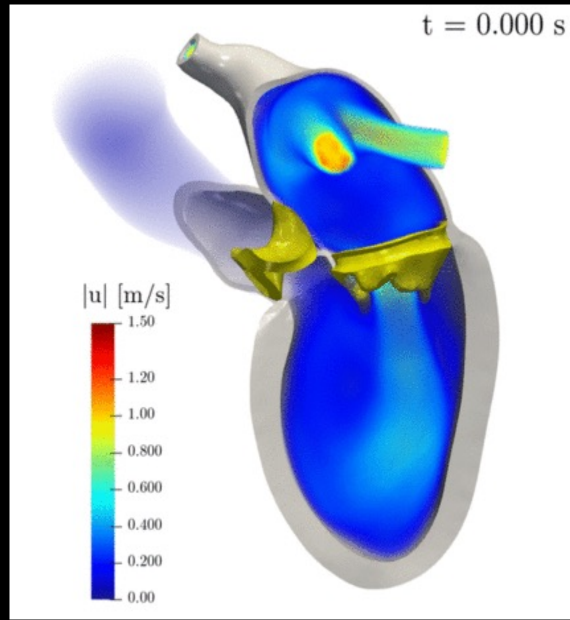
0.6
0.4
0.2
0.0
velocity magnitude



t = 0.000 s
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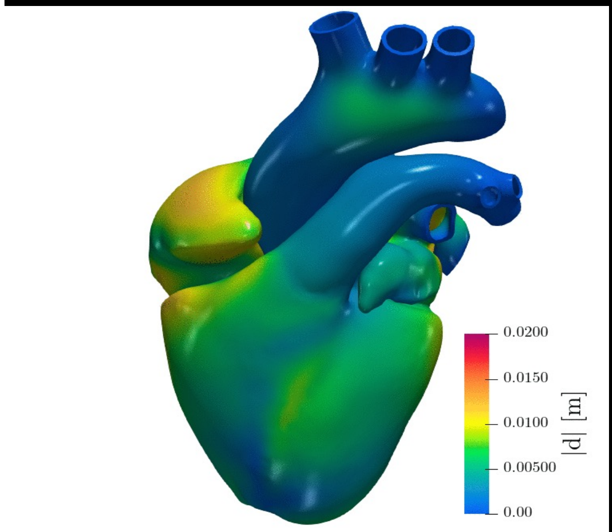

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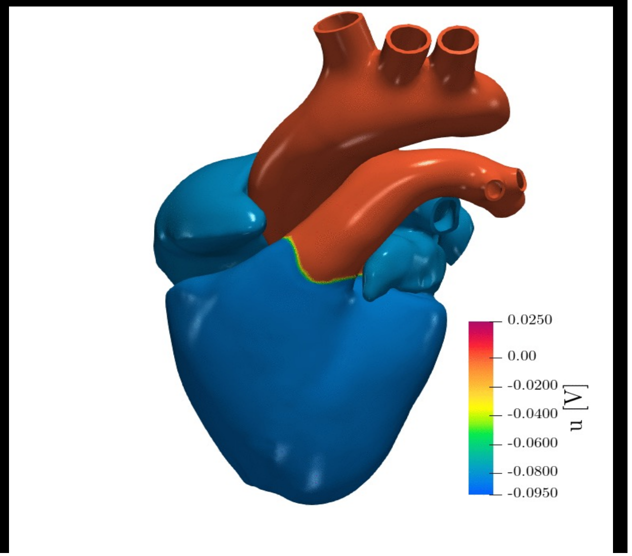

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u [V]

THANK YOU